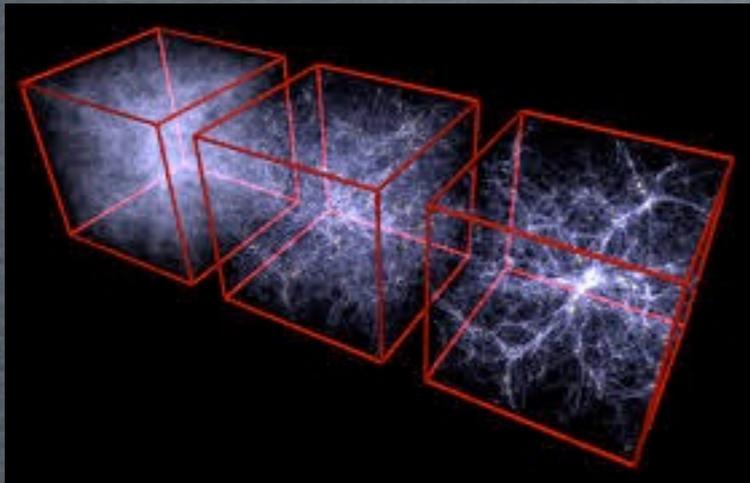


The Bosonic Side of Composite Dark Matter



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INT,
UW, Seattle



Primary collaborators:

Sergey Syritsyn

Ethan Neil

Graham Kribs

Chris Schroeder

Enrico Rinaldi

PRD 88 014502 (2013)

PRD 89 094508 (2013)
(Lattice)

arXiv: 1407.????
(Pheno)



Lattice **S**trong **D**ynamics Collaboration



James Osborn



Evan Berkowitz
Enrico Rinaldi
Chris Schroeder
Pavlos Vranas



Rich Brower
Michael Cheng
Claudio Rebbi
Oliver Witzel
Evan Weinberg



Joe Kiskis



Ethan Neil



David Schaich



Ethan Neil
Sergey Syritsyn



Tom Appelquist
George Fleming
Gennady Voronov



Meifeng Lin



Graham Kribs

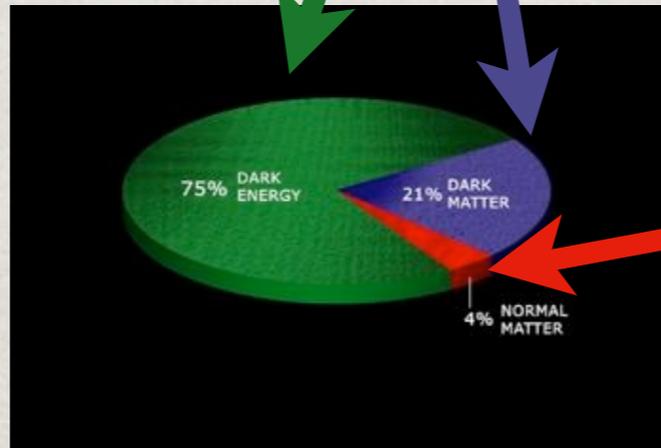


Mike Buchoff

A SLICE OF THE UNIVERSE

???????

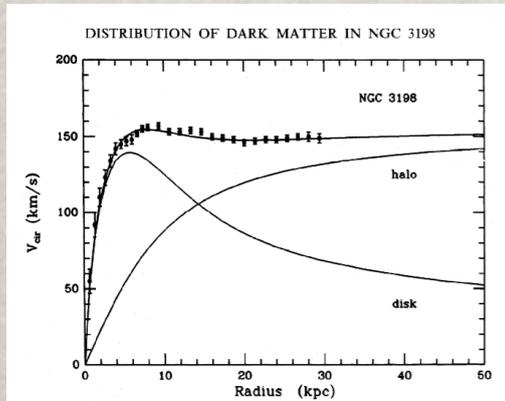
New
Physics!!



**We Are
Here**

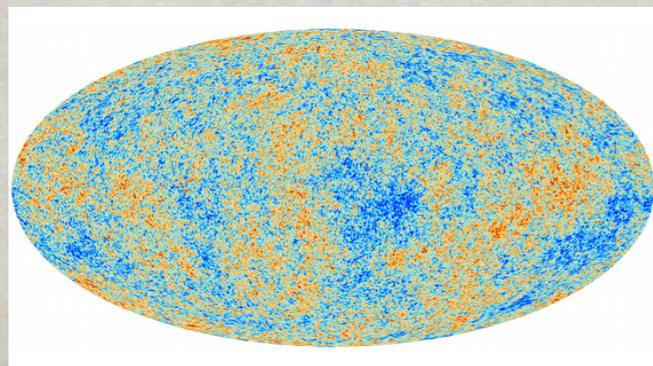
(QCD, EM,
SM, etc.)

How do we know DM is there?



☼ Rotation Curves of Galaxies

☼ Gravitational Lensing



☼ Cosmic Microwave Background

THREE PRIMARY PROPERTIES OF DARK MATTER

Dark Matter Candidate should:

1. Be Long Lived

- Explains why dark matter has survived to today
➔ Implies a new symmetry and/or charge

2. Be EW Charge
Neutral

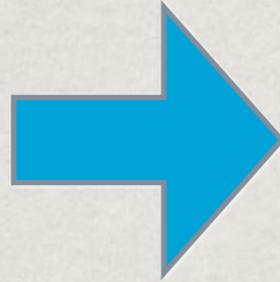
- Explains why there is no visible evidence
➔ Implies lightest stable particle is chargeless

3. Explain Observed
Relic Density

$$\rho_D \sim 0.25 \rho_c$$

THERMAL RELIC

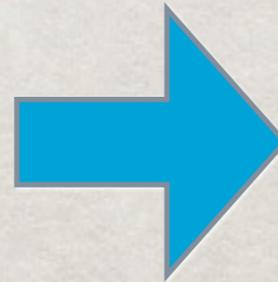
Dark Matter
Annihilates



How much do we
see today?

One approach to DM theories:

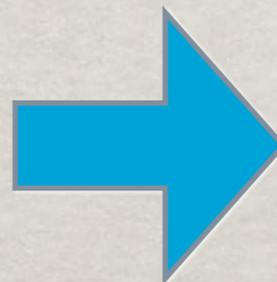
Choose DM Mass
Choose DM Interactions



$$\rho_D \sim 0.25 \rho_c$$

“WIMP Miracle”

Assume Interactions
at/near EW Scale



$$M_D \sim \text{TeV}$$

THERMAL RELIC

Dark Matter
Annihilation

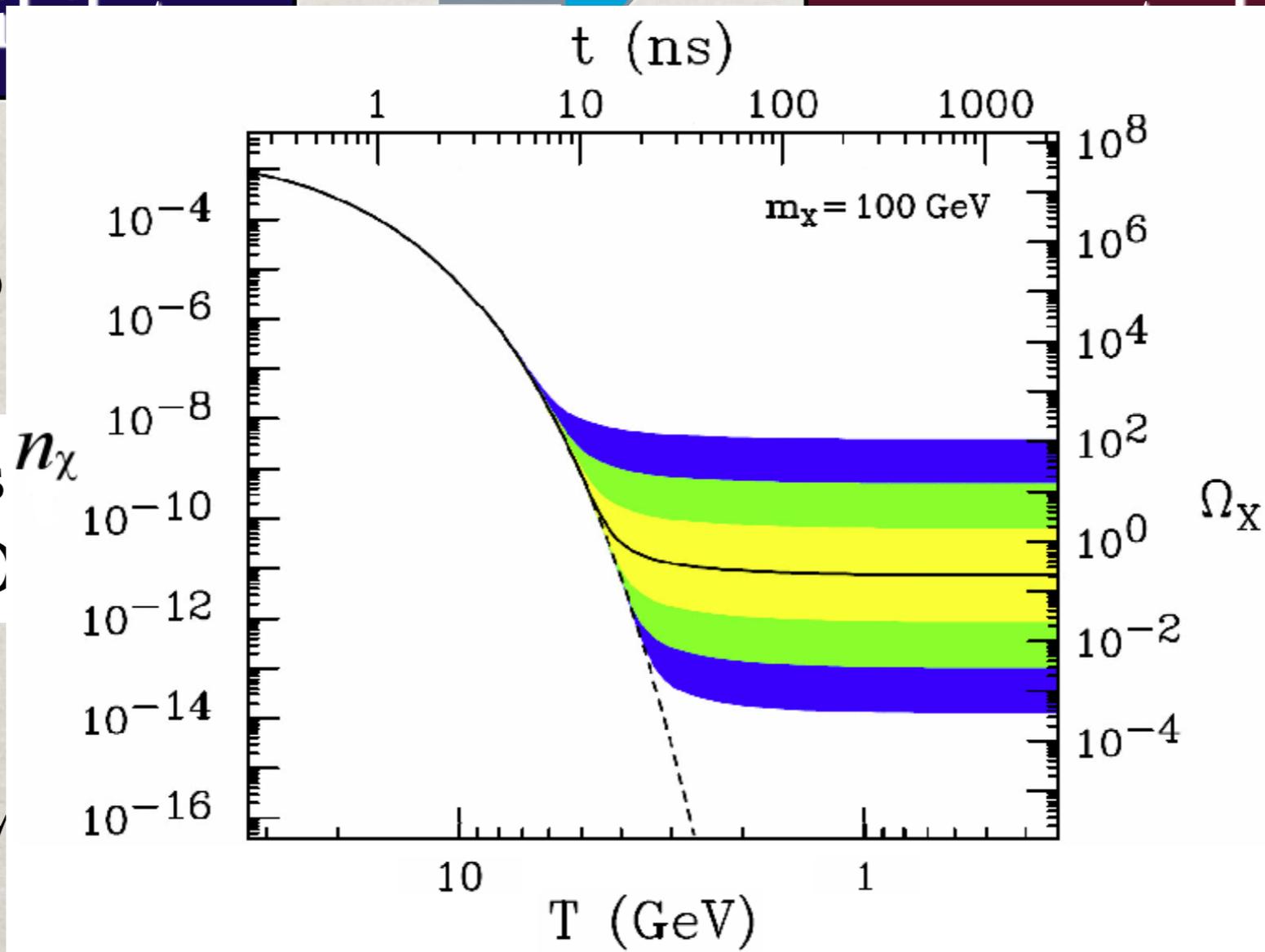


How much do we
produce today?

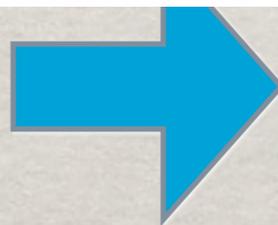
One approach

Choose n_χ
Choose Ω_χ

“WIMP M



Assume Interactions
at/near EW Scale



$M_D \sim \text{TeV}$

AN ASYMMETRIC ALTERNATIVE?

S.Nussinov (1985)

S.M. Barr, R.S.Chivukula, E. Farhi (1990)

R.S.Chivukula, T.P.Walker (1990)

D.B.Kaplan (1992)

Observe a different relation:

$$\rho_D \sim 5\rho_B$$

$$M_D n_D \sim 5M_B n_B$$

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Asymmetry

AN ASYMMETRIC ALTERNATIVE?

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S.M. Barr, R.S.Chivukula, E. Farhi (1990)

R.S.Chivukula, T.P.Walker (1990)

D.B.Kaplan (1992)

Observe a different relation:

$$\rho_D \sim 5 \rho_B \quad \text{Asymmetry}$$
$$M_D n_D \sim 5 M_B n_B$$

If DM density is thermal:

Unjustified Accident

Natural if DM density is also tied to asymmetry

$$n_D \sim n_B \quad \longrightarrow \quad M_D \sim 5 \text{ GeV}$$

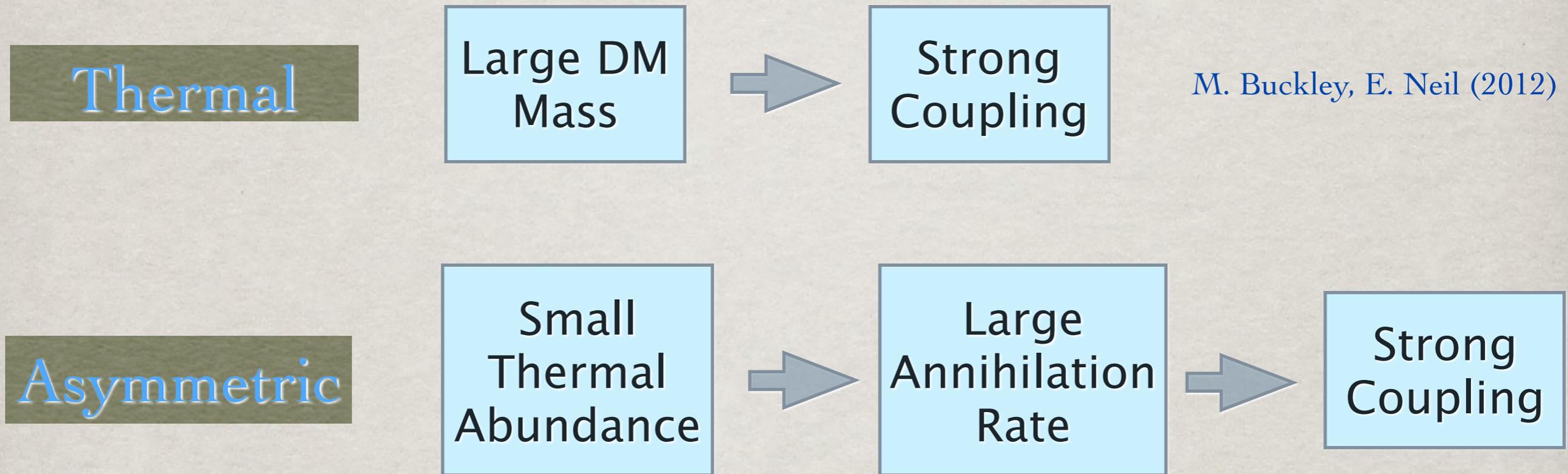
$$M_D \gg M_B \quad \longrightarrow \quad n_B \gg n_D \sim e^{-M_D/T_{sph}}$$

Sphaleron
connection



Direct or Indirect
coupling to EW

WHY STRONG COUPLING?



Studying strongly-coupled composite systems critical to fully understand landscape of DM theories

...this is where the lattice can play significant role!

THREE PRIMARY PROPERTIES OF DARK MATTER

Dark Matter Candidate should:

1. Be Long Lived

➔ Implies a new symmetry and/or charge

Example: Baryons - Baryon Number

Mesons - G-parity [Y.Bai, R.J.Hill \(2010\)](#)

2. Be EW Charge
Neutral

➔ Implies lightest stable particle is chargeless

Example: Can form neutral baryons

3. Explain observed
relic density

➔ Asymmetry require charge couplings

Example: Charged Constituents

LONG TERM OBJECTIVE

ULTIMATE GOAL:

To place a lower bound on nuclear cross-sections of composite DM with charged constituents

We Want:

- ★ Bound general classes of composite DM from first principles
- ★ Explore **Higgs exchange** and **EM moments** for direct detection
- ★ Study classes of models with minimal SM interaction strength

Final Goal:



Polarizabilities

LONG TERM OBJECTIVE

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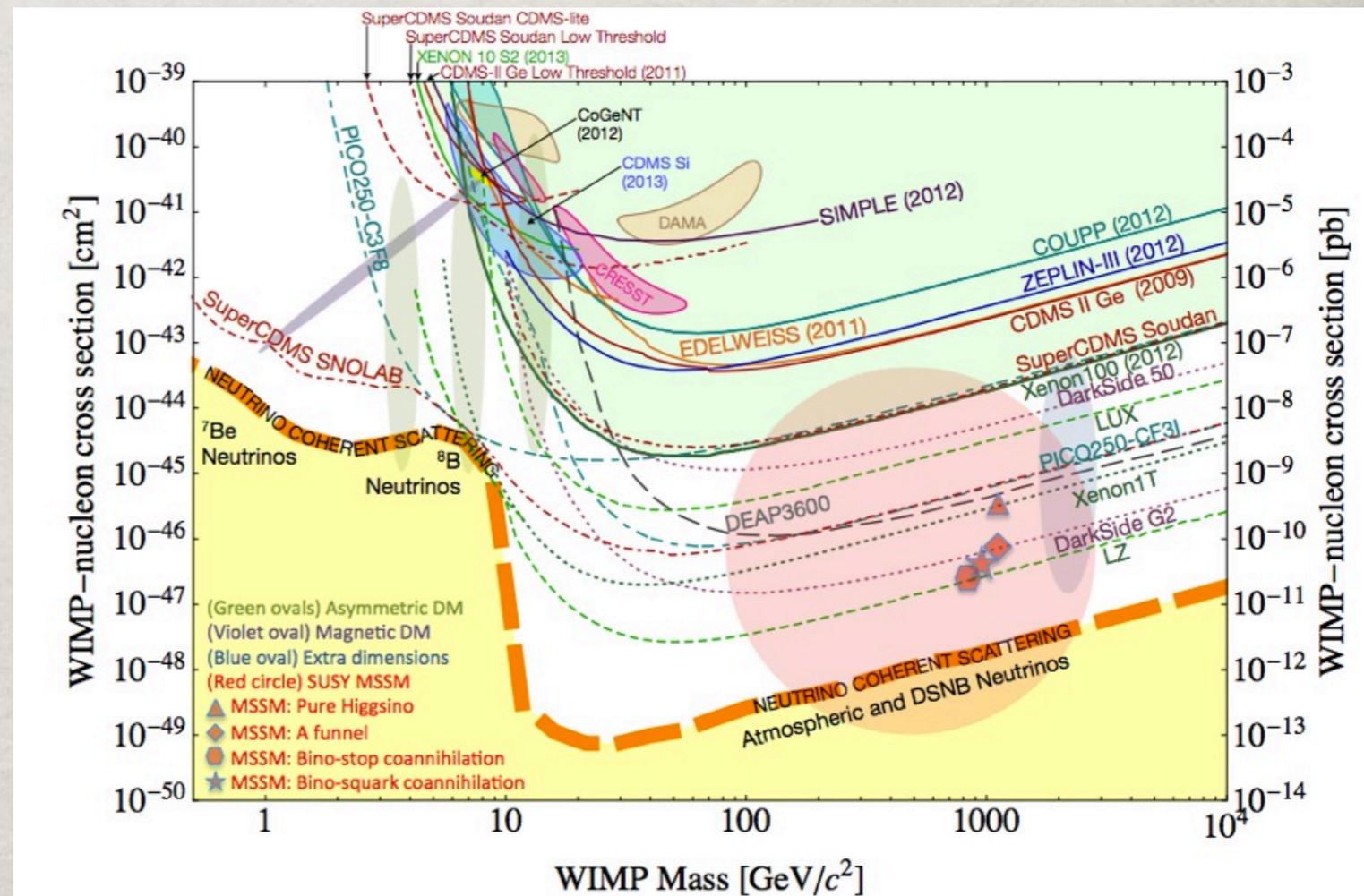
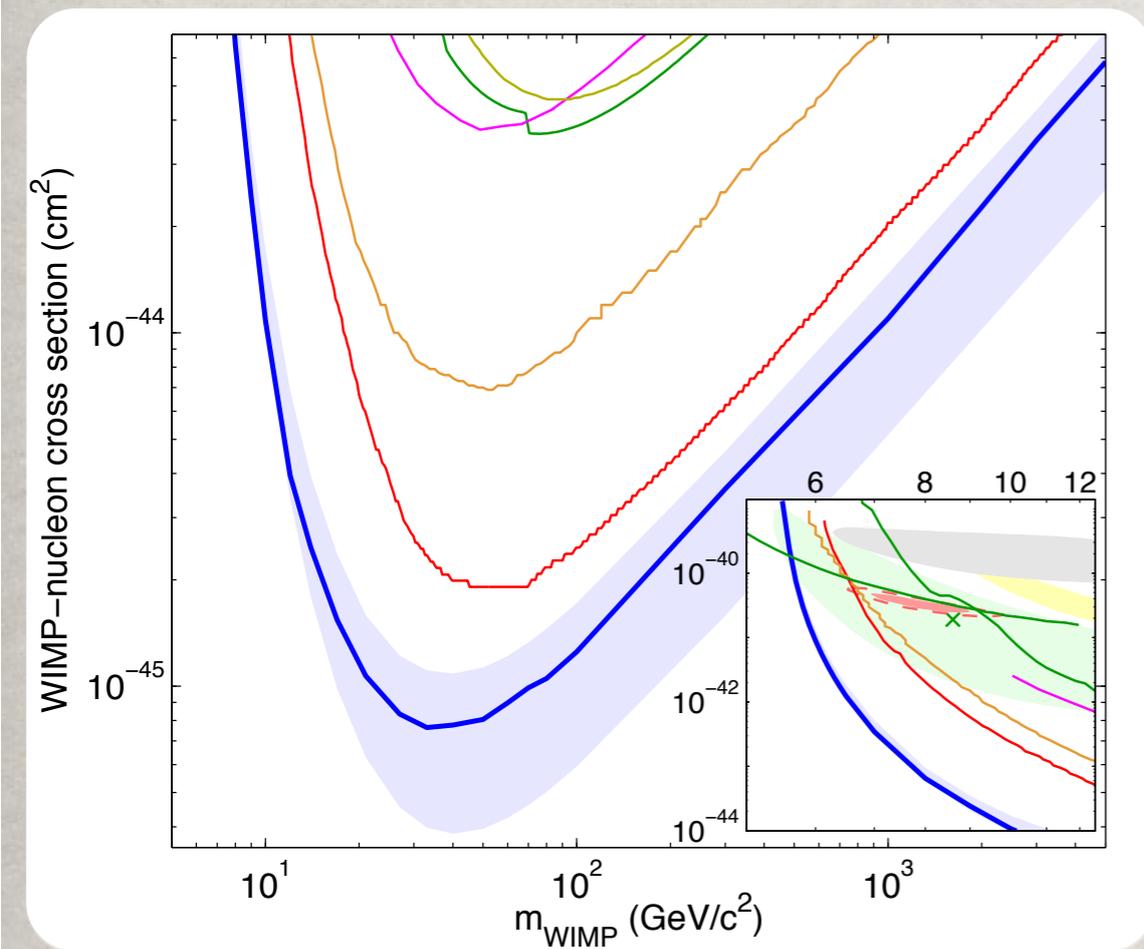
Polarizabilities

OUR FOCUS: DIRECT DETECTION

☼ Before asking any other question, how strong are direct detection bounds?

LUX: PRL 112.091303

Experimental DM Frontier



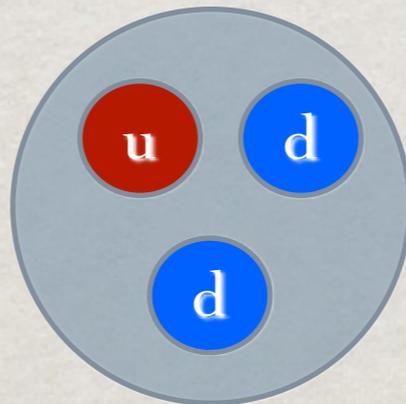
Spin-independent (coherent) - Very tight constraints

$$\sigma \lesssim 10^{-45} \text{ cm}^2$$

BARYON FLAVOR SYMMETRY

★ Flavor Non-symmetric

Example: (3-color neutron ala QCD)



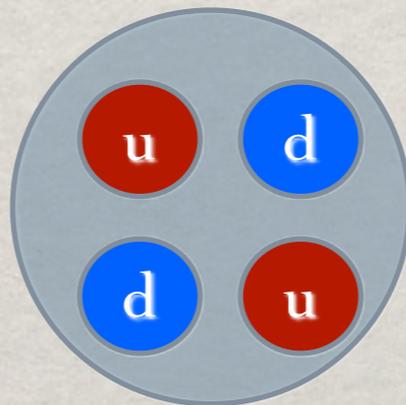
$$Q_u = Q_d$$

or

$$Q_u \neq Q_d$$

★ Flavor Symmetric

Example: (4-color neutron)



$$Q_u = -Q_d$$

only

HOW WE MIGHT SEE IT?

Dim-5

$$\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}$$

Magnetic
Moment

Dim-6

$$(\bar{\psi}\psi)v_{\mu}\partial_{\nu}F^{\mu\nu}$$

Charge
Radius

Dim-7

$$(\bar{\psi}\psi)F_{\mu\nu}F^{\mu\nu}$$

Polarizability

Odd Nc

No baryon flavor sym.



Odd Nc

Baryon flavor sym.



Even Nc

No Baryon flavor sym.



Even Nc

Baryon flavor sym.



FOCUS OF PREVIOUS WORK

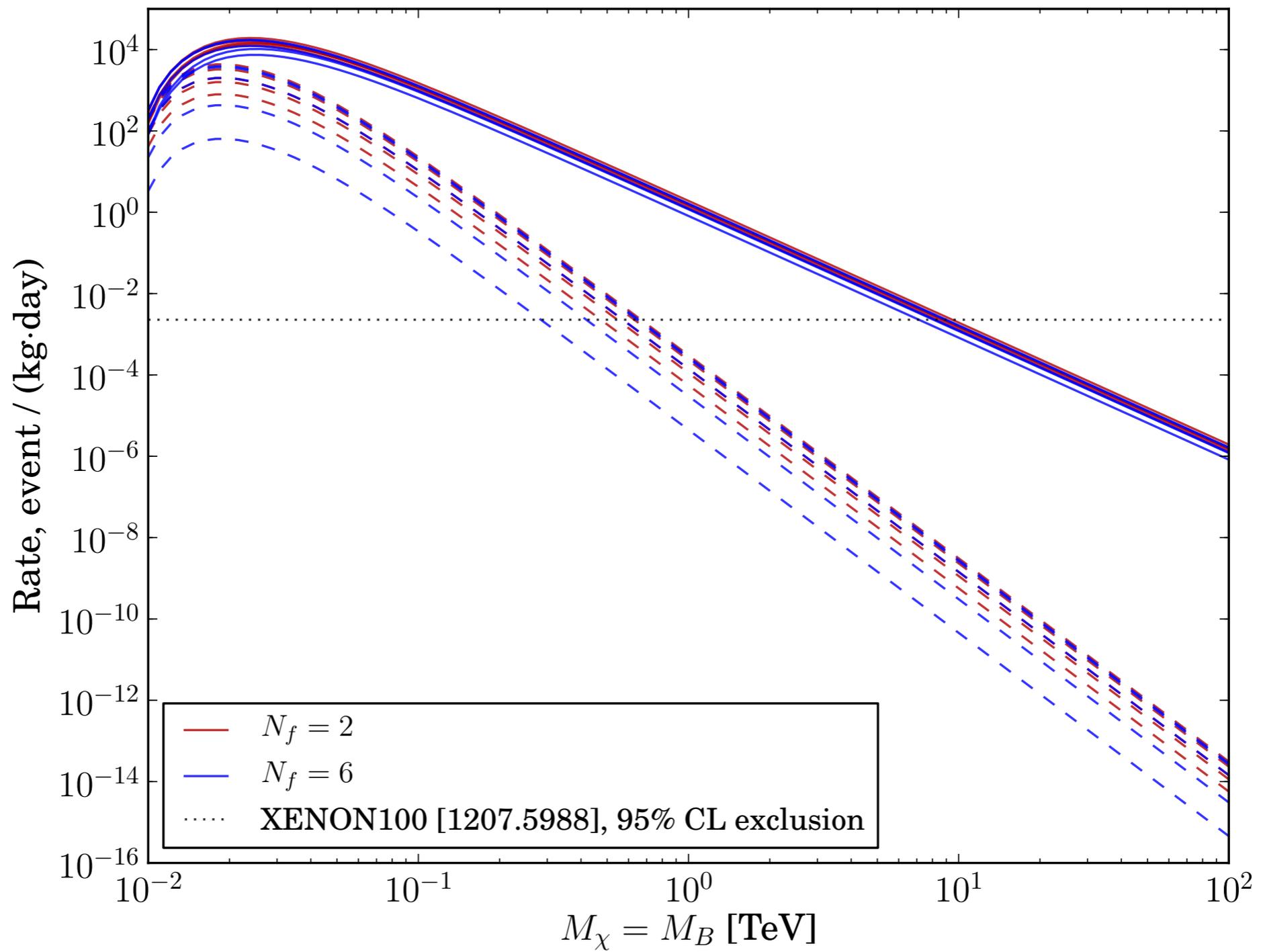
☀ Direct detection exclusions for odd number of colors

Explore:

- ✿ 3-colors
- ✿ Multiple degenerate masses
- ✿ 2 and 6 light flavors

Explores a range of confining theories for odd N_c theory

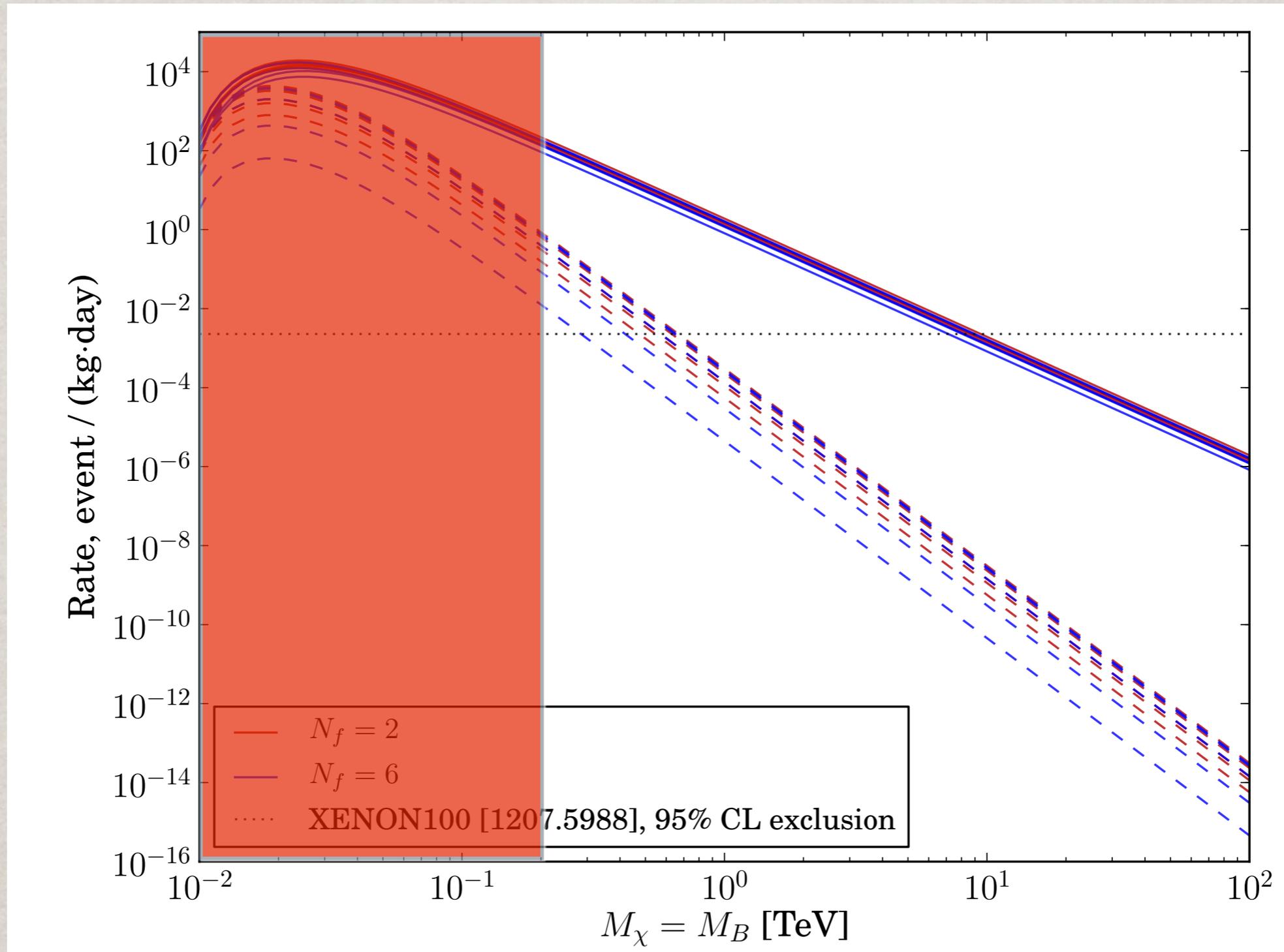
EXCLUSION PLOTS



Dashed horizontal line - Xenon100

PRD 88 014502 (2013)

EXCLUSION PLOTS



Dashed horizontal line - Xenon100
PRD 88 014502 (2013)

LEP Bound
on charged
particles:
 $M > 88$ GeV

FOCUS OF RECENT WORK

☀ Direct detection exclusions for even number of colors

Explore:

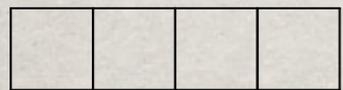
- ✿ 4-colors
- ✿ Multiple degenerate masses (quenched)
- ✿ Baryon spectra and sigma term

Allows for cross-section bounds from Higgs exchange

4-COLOR BARYONS

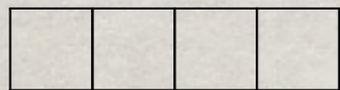
☀ Bosonic baryons

One Flavor: U

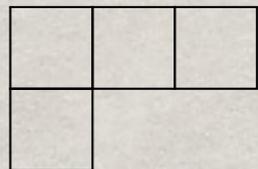


Spin-2: $\mathcal{O}_{B,S2}^{N_F=1} = (U^T C \gamma^i U)(U^T C \gamma^j U) \quad i \neq j$

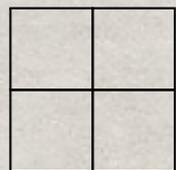
Two Flavors: $U \quad D$



Spin-2: $\mathcal{O}_{B,S2}^{N_F=2} = (U^T C \gamma^i U)(U^T C \gamma^j U) \quad i \neq j$



Spin-1: $\mathcal{O}_{B,S1}^{N_F=2} = (U^T C \gamma^i U)(U^T C \gamma^5 D)$



Spin-0: $\mathcal{O}_{B,S1}^{N_F=2} = (U^T C \gamma^5 D)(U^T C \gamma^5 D)$

HIGGS EXCHANGE

✻ Higgs-nucleon cross-section:

$$\sigma_0(B, n) = \frac{\mu(m_B, m_n)^2}{\pi A^2} (Z\mathcal{M}_p + (A - Z)\mathcal{M}_n)^2$$

$$\mathcal{M}_a = \frac{y_f y_q}{2m_h^2} \sum_f \langle B | \bar{f} f | B \rangle \sum_q \langle a | \bar{q} q | a \rangle$$

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Per Nucleon \nearrow

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SM:

$$\text{Light Quarks: } \langle n | m_q \bar{q} q | n \rangle = m_n f_q^{(n)}$$

$$\text{Heavy Quarks: } \langle n | m_q \bar{q} q | n \rangle = \frac{2}{27} m_n \left(1 - \sum_{q=u,d,s} f_q^{(n)} \right)$$

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Per Nucleon

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Lattice
QCD

HIGGS EXCHANGE

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Dark:

$$\frac{1}{\sqrt{2}} y_f \equiv \left. \frac{\partial m_f(h)}{\partial h} \right|_{h=v} \quad \text{(Feynman-Hellmann)} \quad f_f^B = \frac{\langle B | m_f \bar{f} f | B \rangle}{m_B} = \frac{m_f}{m_B} \frac{\partial m_B}{\partial m_f}$$

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BSM mass couplings
(Perturbative)

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BSM mass couplings
(Perturbative)

Strong Dynamics
(Non-perturbative)
Robust lattice results

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$$\mathcal{M}_{p,n} = \frac{g_{p,n}g_B}{m_h^2}$$

SM:

$$g_{p,n} = \frac{m_{p,n}}{v} \left[\sum_{q=u,d,s} f_q^{(p,n)} + \frac{6}{27} \left(1 - \sum_{q=u,d,s} f_q^{(p,n)} \right) \right]$$

Dark:

$$g_B = \frac{m_B}{v} \sum_f \frac{v}{m_f} \left. \frac{\partial m_f(h)}{\partial h} \right|_{h=v} f_f^{(B)}$$

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Lattice
QCD

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Dark:

$$g_B = \frac{m_B}{v} \sum_f \overbrace{\frac{v}{m_f} \frac{\partial m_f(h)}{\partial h}}^{\alpha_f} \Big|_{h=v} f_f^{(B)}$$

Lattice
DM

CALCULATION DETAILS

28 quenched Ensembles:

- Two # colors
- Four lattice volumes
- Three lattice spacings
- 3-6 fermion masses

N_c	β	κ	$N_s^3 \times N_t$	# Meas.
4	11.028	0.1554	$16^3 \times 32$	4878
			$32^3 \times 64$	1126
		0.15625	$16^3 \times 32$	4765
			$32^3 \times 64$	1146
			$48^3 \times 96$	1091
	11.5	0.1572	$32^3 \times 64$	1075
			0.1515	$16^3 \times 32$
				$32^3 \times 64$
		0.1520	$16^3 \times 32$	2872
			$32^3 \times 64$	1052
		0.1523	$16^3 \times 32$	2976
			$32^3 \times 64$	914
			$48^3 \times 96$	637
			$64^3 \times 128$	489
		0.1524	$16^3 \times 32$	2970
			$32^3 \times 64$	863
		0.1527	$32^3 \times 64$	1011
			12.0	0.1475
		0.1480		
		0.1486	$32^3 \times 64$	1055
$32^3 \times 64$	1050			
$48^3 \times 96$	1150			
$64^3 \times 128$	928			
		0.1495	$32^3 \times 64$	1043
				0.1496
3	6.0175	0.1537		
			0.1547	$32^3 \times 64$

Table 1: Ensembles and number of measurements

CALCULATION DETAILS

28 quenched Ensembles:

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- 3-6 fermion masses

Summary of Lattice Details:

1. Volume systematic within statistical errors



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Table 1: Ensembles and number of measurements

CALCULATION DETAILS

28 quenched Ensembles:

- Two # colors
- Four lattice volumes
- Three lattice spacings
- 3-6 fermion masses

Summary of Lattice Details:

1. Volume systematic within statistical errors
2. Discretization systematic within statistical errors



N_c	β	κ	$N_s^3 \times N_t$	# Meas.
4	11.028	0.1554	$16^3 \times 32$	4878
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		0.1523	$16^3 \times 32$	2976
			$32^3 \times 64$	914
			$48^3 \times 96$	637
			$64^3 \times 128$	489
		0.1524	$16^3 \times 32$	2970
			$32^3 \times 64$	863
	12.0	0.1527	$32^3 \times 64$	1011
			0.1475	$32^3 \times 64$
		0.1480		$32^3 \times 64$
			0.1486	$32^3 \times 64$
		0.1491		$16^3 \times 32$
			$32^3 \times 64$	1050
			$48^3 \times 96$	1150
			$64^3 \times 128$	928
		0.1495	$32^3 \times 64$	1043
			0.1496	$32^3 \times 64$
3	6.0175	0.1537		$32^3 \times 64$
			0.1547	$32^3 \times 64$

Table 1: Ensembles and number of measurements

CALCULATION DETAILS

28 quenched Ensembles:

- Two # colors
- Four lattice volumes
- Three lattice spacings
- 3-6 fermion masses

Summary of Lattice Details:

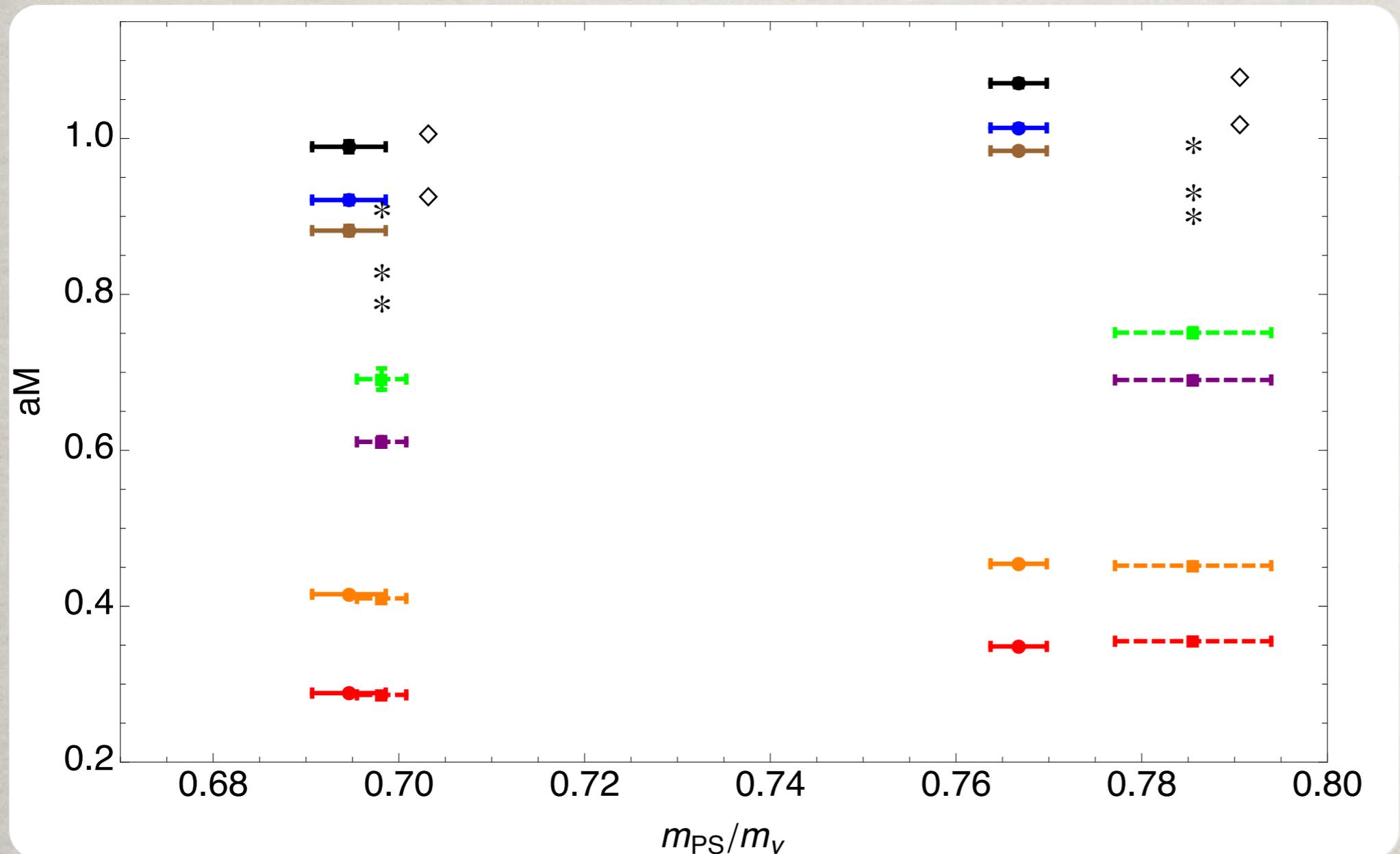
1. Volume systematic within statistical errors
2. Discretization systematic within statistical errors
3. Three points to extract slope (more would be preferred)



N_c	β	κ	$N_s^3 \times N_t$	# Meas.
4	11.028	0.1554	$16^3 \times 32$	4878
			$32^3 \times 64$	1126
		0.15625	$16^3 \times 32$	4765
			$32^3 \times 64$	1146
			$48^3 \times 96$	1091
	11.5	0.1572	$32^3 \times 64$	1075
			0.1515	$16^3 \times 32$
				$32^3 \times 64$
		0.1520	$16^3 \times 32$	2872
			$32^3 \times 64$	1052
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LARGE N COMPARISONS



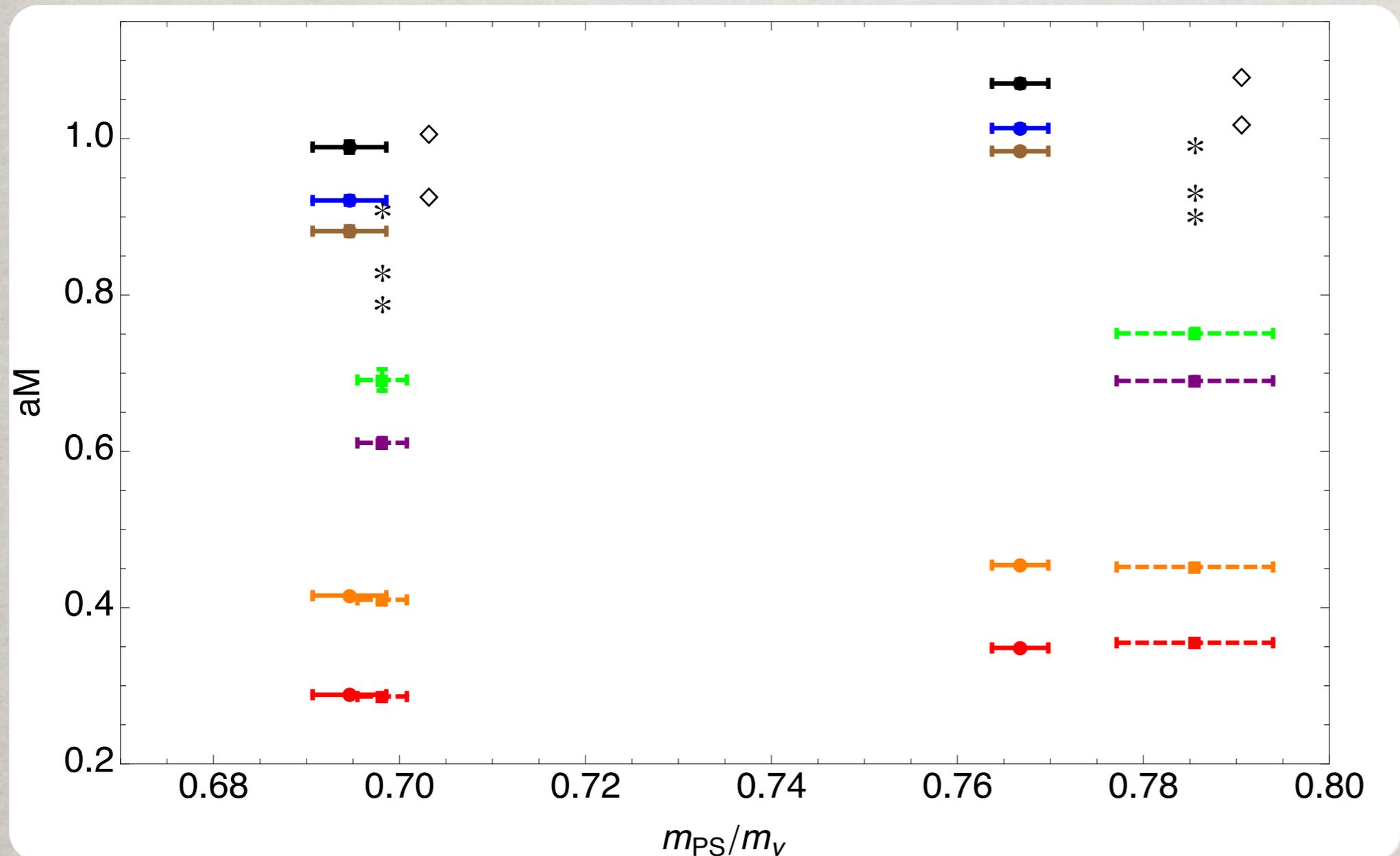
Solid - 4 colors
Dashed - 3 colors

Black - Spin 2
Blue - Spin 1
Brown - Spin 0
Green - Spin 3/2
Purple - Spin 1/2
Orange - Vector
Red - Pseudoscalar

$$* : M(N_c, J) = N_c m_0 + \frac{J(J+1)}{N_c} B + \mathcal{O}(1/N_c^2)$$

$$\diamond : M(N_c, J) = N_c m_0^{(0)} + C + \frac{J(J+1)}{N_c} B + \mathcal{O}(1/N_c^2)$$

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Key Observation
from DeGrand (2013)

SCALE SETTING

How do we define lattice spacing in physical units?

Lattice QCD: Hadron Masses, HQ potentials, etc.

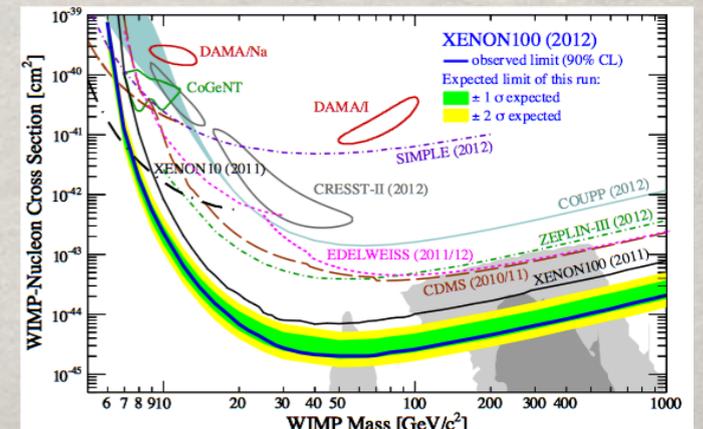
(Example) $aM_\Omega = \#$ \longrightarrow $a \approx \frac{\#}{1670 \text{ MeV}}$

Technicolor: "Higgs" vev

$a f_\pi \xrightarrow{m_f \rightarrow 0} \#$ \longrightarrow $a \approx \frac{\#}{246 \text{ GeV}}$

Dark Matter: Dark Matter Mass

$aM_B = \#$ \longrightarrow $a \approx \frac{\#}{M_B}$



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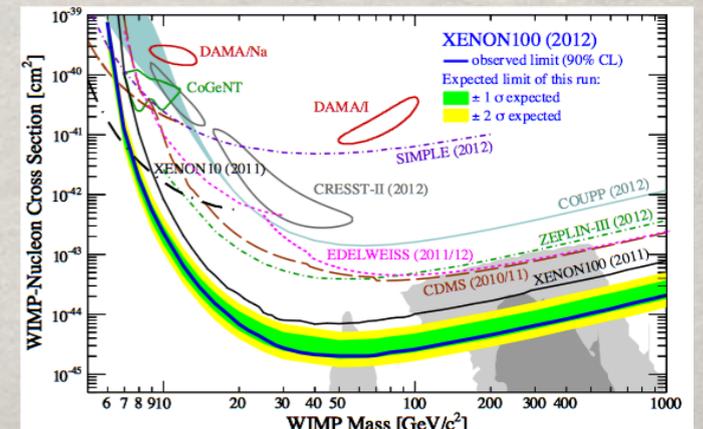
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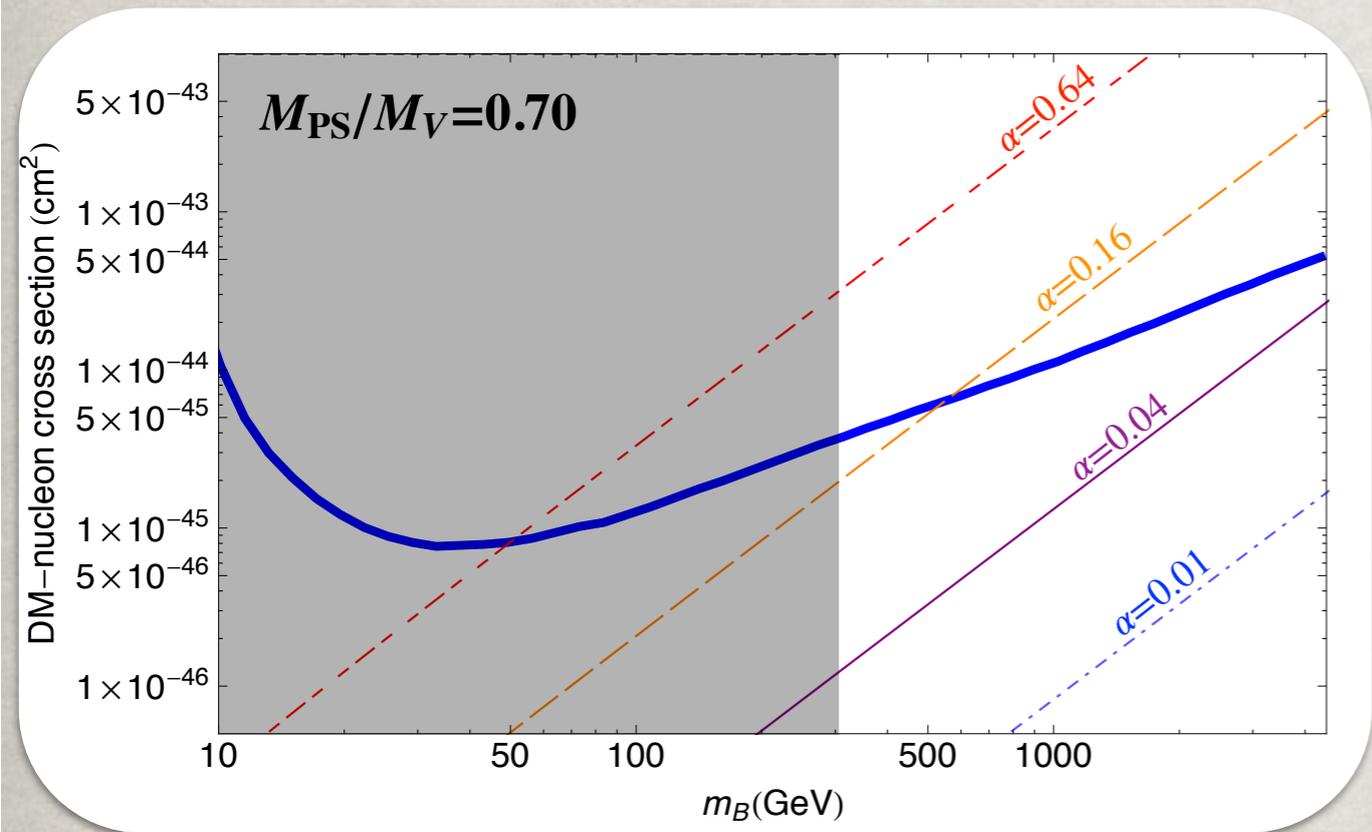
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Vary this value \nearrow



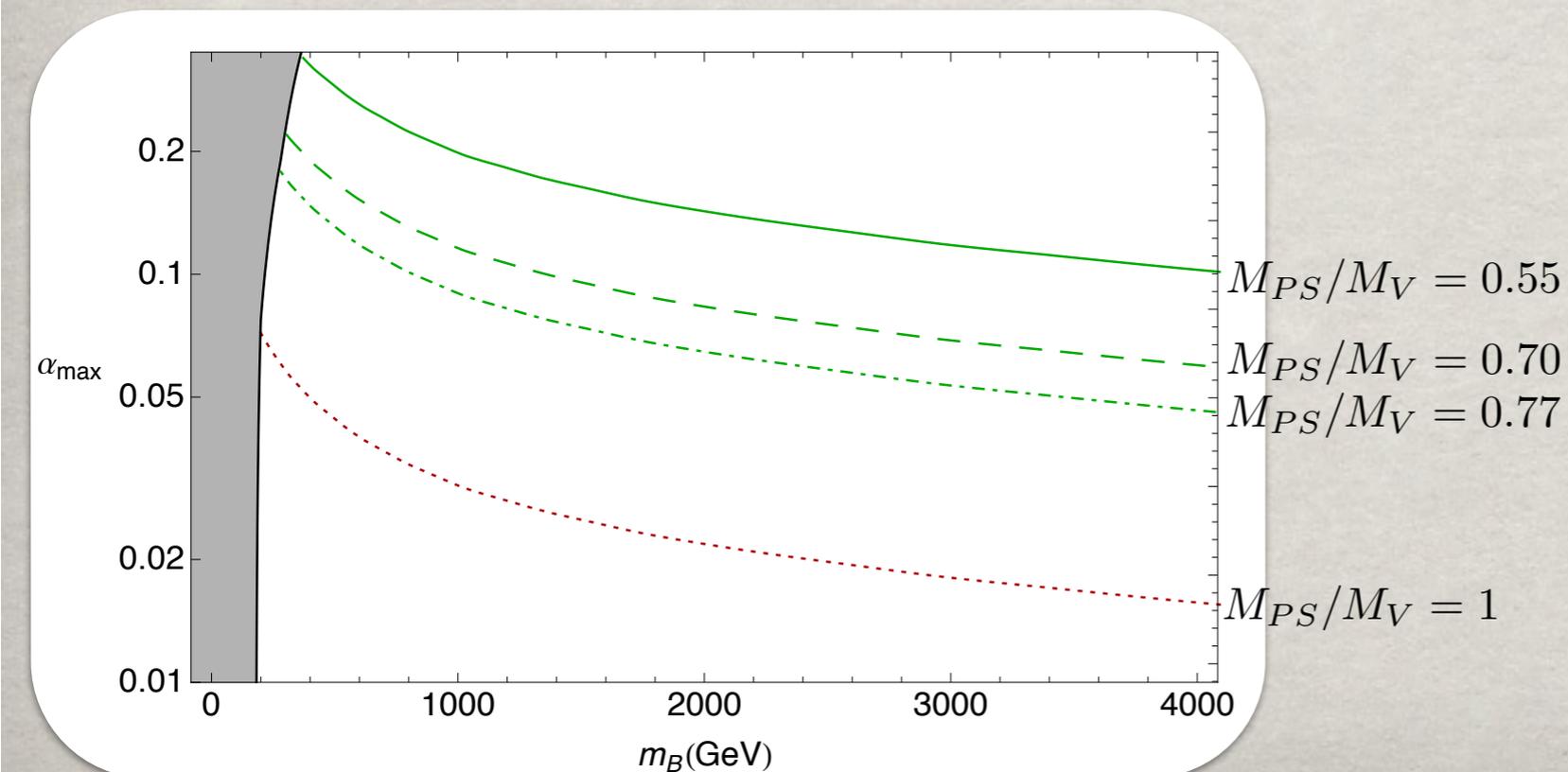
SIGMA TERM & HIGGS BOUND



$$\alpha \equiv \left. \frac{v}{m_f} \frac{\partial m_f(h)}{\partial h} \right|_{h=v}$$

$$0.153 \lesssim f^{(B)} \lesssim 0.338$$

$$2.82 \lesssim \frac{m_B}{m_{PS}} \lesssim 3.71$$



LEP Bound
on charged
particles:
 $M > 88 \text{ GeV}$

PARTICULAR MODEL

✻ Four Dirac Flavors with vector-like masses

Field	$SU(N)_D$	$(SU(2)_L, Y)$	Q
$F_1 = \begin{pmatrix} F_1^u \\ F_1^d \end{pmatrix}$	\mathbf{N}	$(\mathbf{2}, 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
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Mass Matrix (custodial symmetric):

$$\mathcal{L}_M = (\bar{\psi}_A^u \bar{\psi}_B^u) \begin{pmatrix} M - \Delta & yv/\sqrt{2} \\ yv/\sqrt{2} & M + \Delta \end{pmatrix} \begin{pmatrix} \psi_A^u \\ \psi_B^u \end{pmatrix} + (\bar{\psi}_A^d \bar{\psi}_B^d) \begin{pmatrix} M - \Delta & yv/\sqrt{2} \\ yv/\sqrt{2} & M + \Delta \end{pmatrix} \begin{pmatrix} \psi_A^d \\ \psi_B^d \end{pmatrix}$$

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$$\sin^2 \theta = \frac{1}{2} \left(1 - \frac{2\Delta}{\sqrt{2y^2v^2 + 4\Delta^2}} \right)$$

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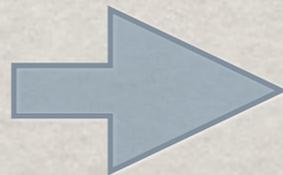
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$$\alpha \approx \frac{yv}{M} \quad M \gg 2yv \gg \Delta$$

Linear

$$\alpha \approx \frac{2(yv)^2}{M\Delta} \quad M \gg \Delta \gg 2yv$$

Quadratic

CROSS SECTION SUMMARY

✻ Back to cross section:

$$\sigma_0(B, n) = \frac{\mu(m_B, m_n)^2}{\pi A^2} (Z\mathcal{M}_p + (A - Z)\mathcal{M}_n)^2$$

$$\mathcal{M}_{p,n} = \frac{g_{p,n}g_B}{m_h^2}$$

$$g_B = \left(\frac{m_B}{v}\right) \alpha f^{(B)}$$

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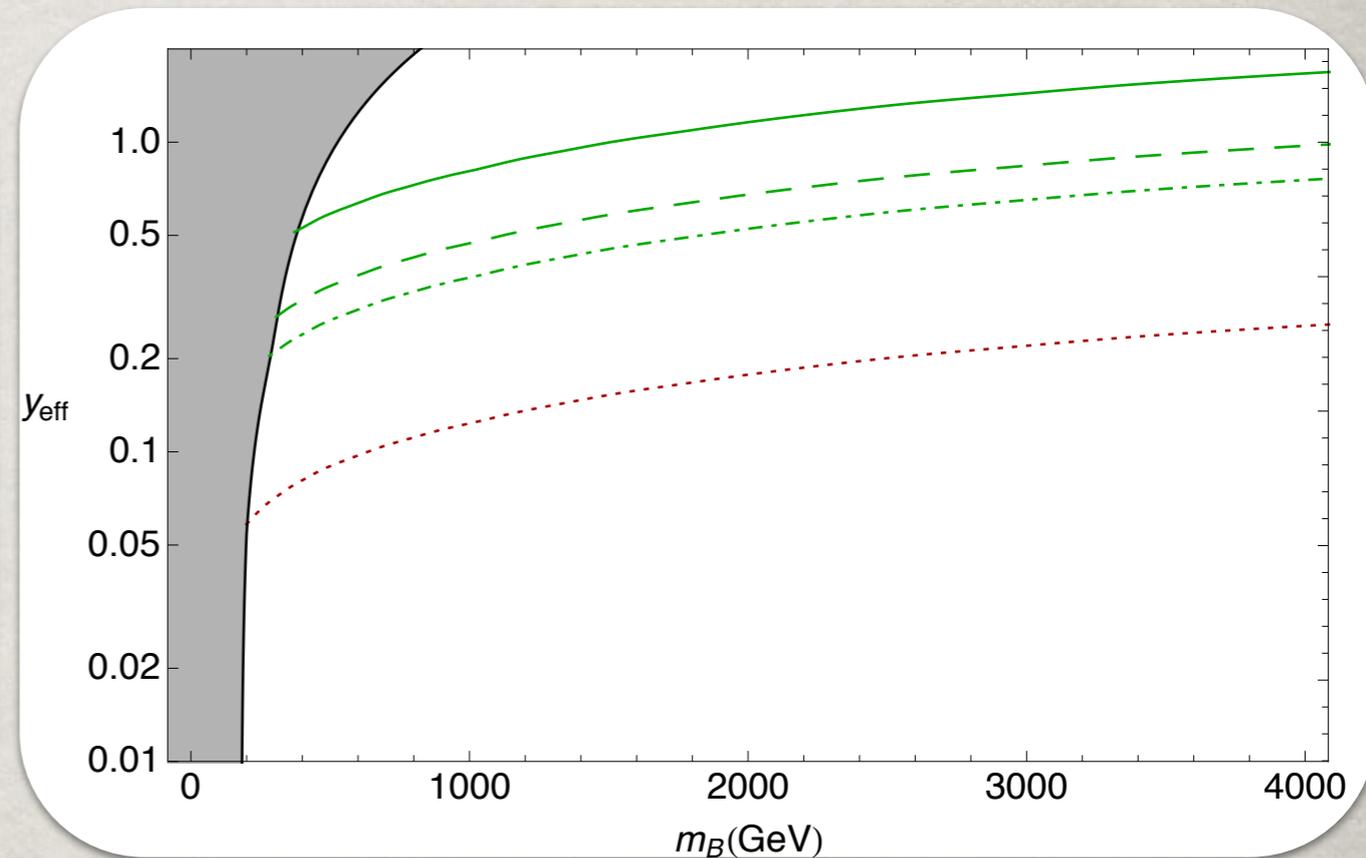
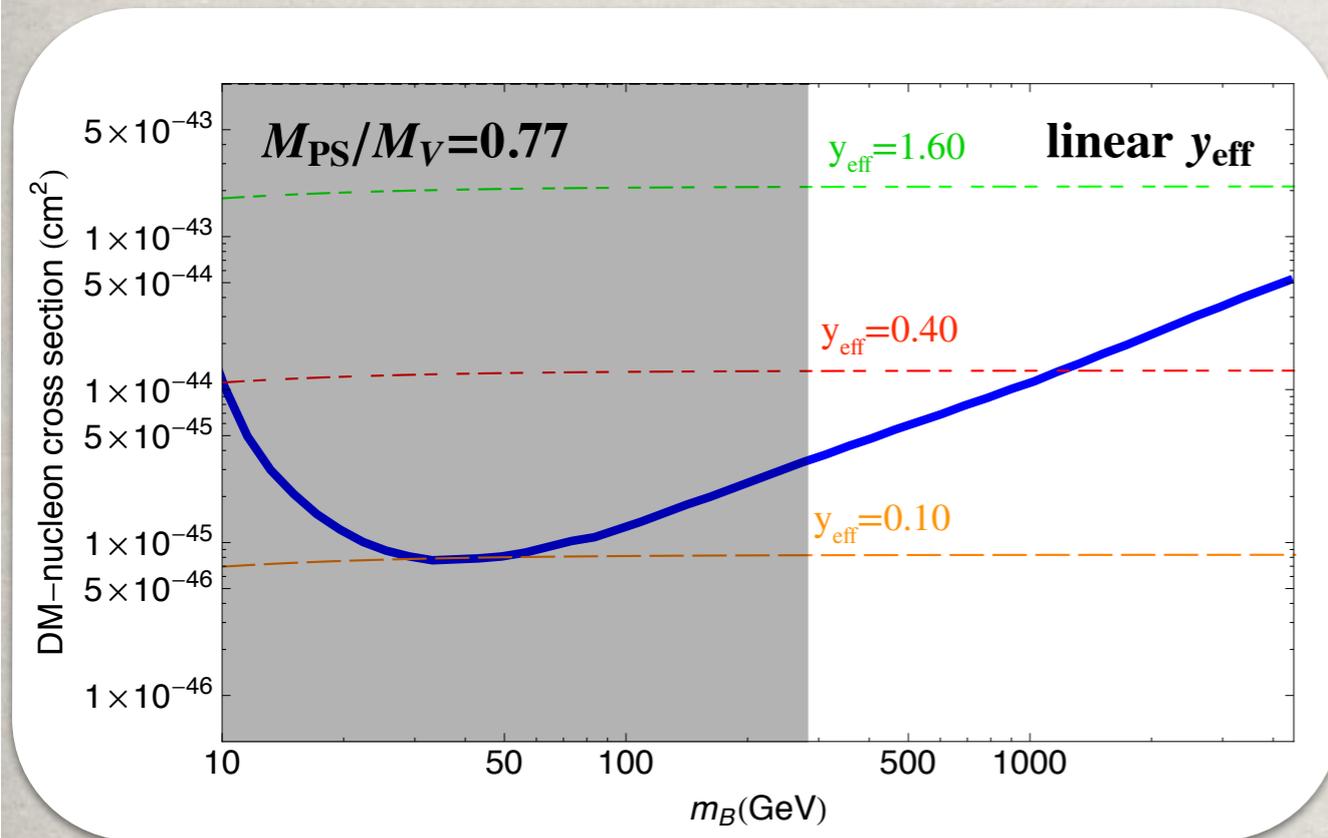
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Varies with scale setting

Varies with model parameters

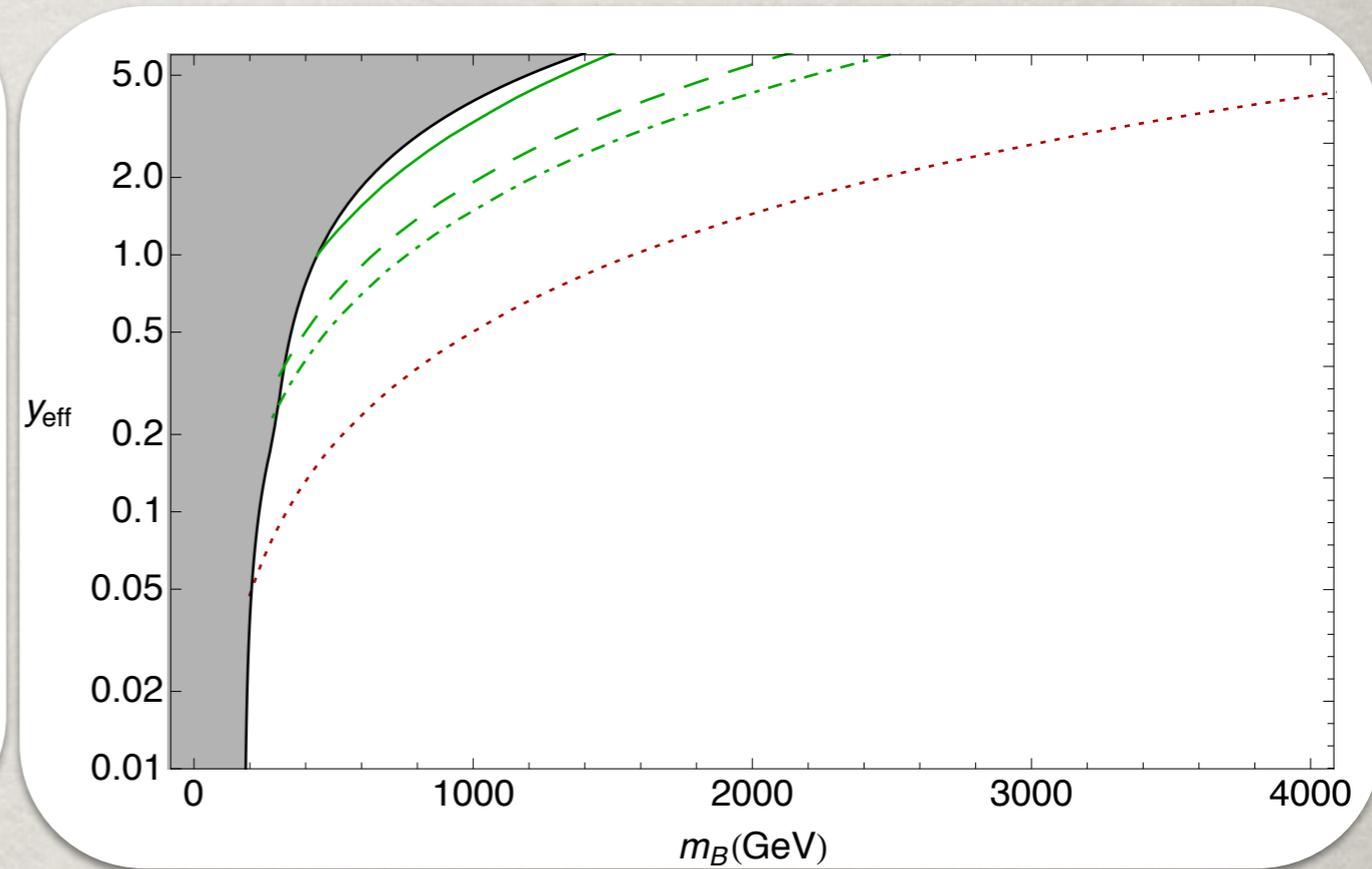
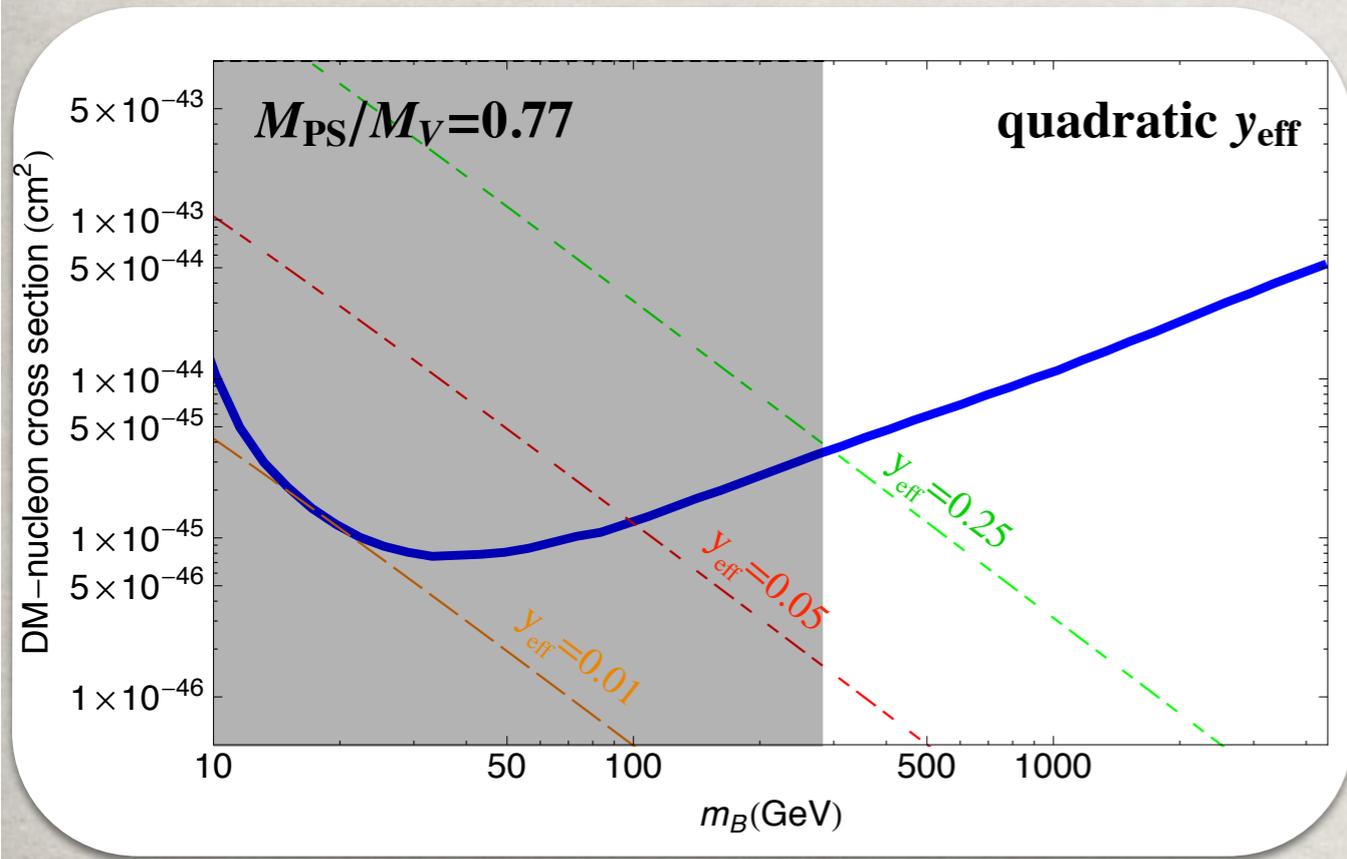
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LINEAR REGIME BOUNDS



$$y_{\text{eff}} \equiv y \left(\frac{M_B}{M} \right) \approx \alpha \frac{M_B}{v}$$

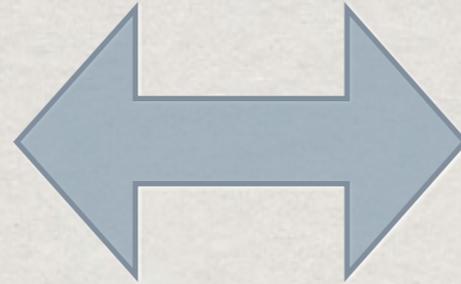
QUADRATIC REGIME BOUNDS



$$y_{\text{eff}}^2 \equiv y^2 \left(\frac{M_B^2}{M\Delta} \right) \approx \alpha \frac{M_B^2}{v^2}$$

DIAGRAMMATIC SUMMARY

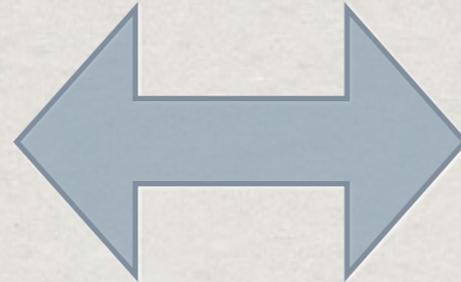
Strongly coupled DM
motivated from relic density



Composite DM addresses
stability, neutrality, and density

DIAGRAMMATIC SUMMARY

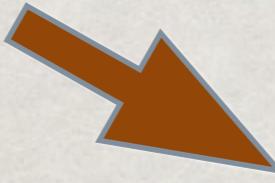
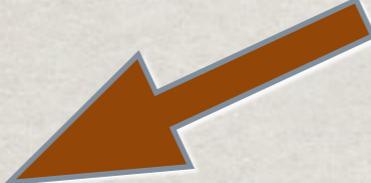
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Our Focus: Direct Detection
of composites with charged constituents

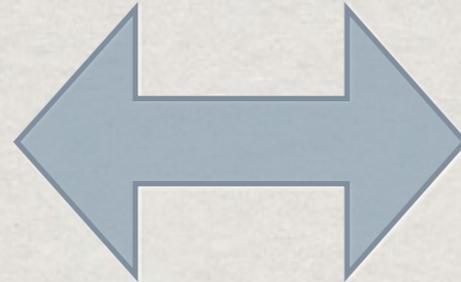
Fermion



Bosons

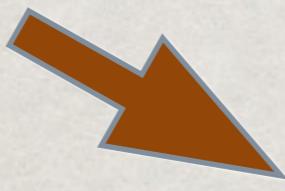
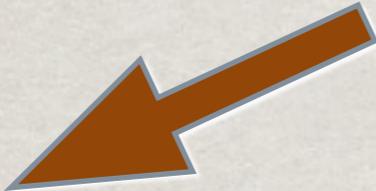
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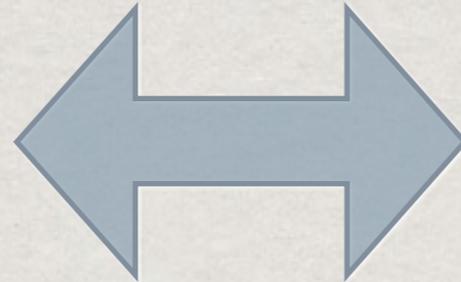
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Dominant interaction:
Magnetic Moment
Charge Radii

$$M_{DM} > 10 \text{ TeV}$$

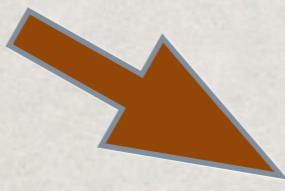
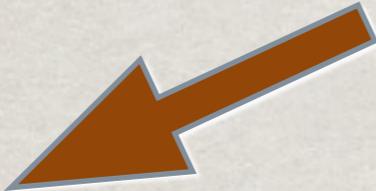
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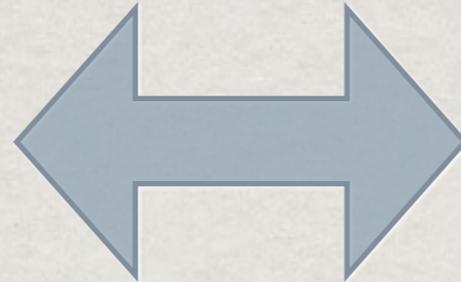
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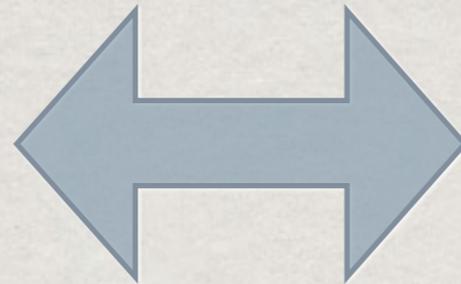
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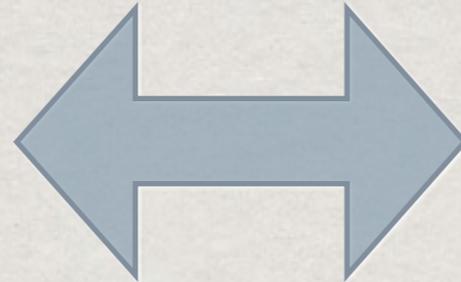
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Vector-like masses

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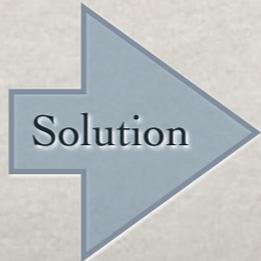
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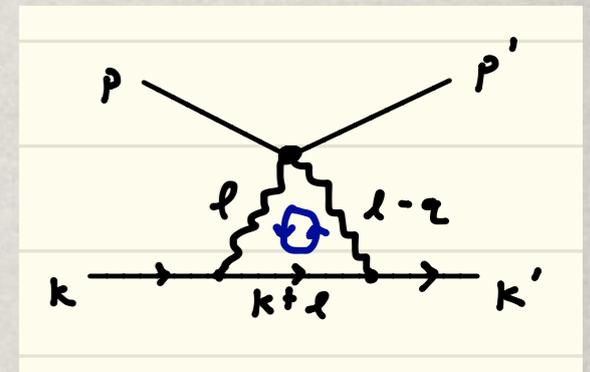
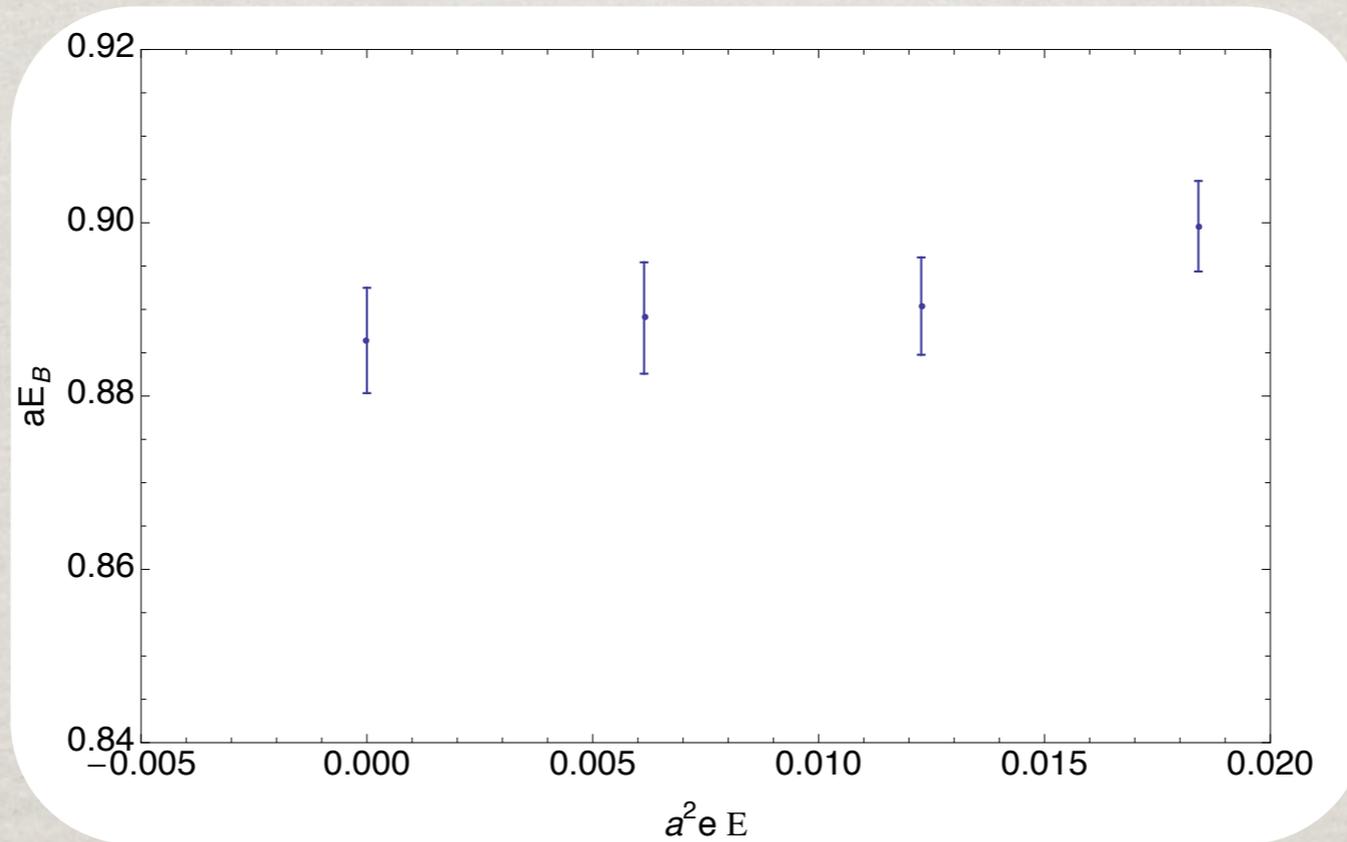


Vector-like masses

VERY PRELIMINARY

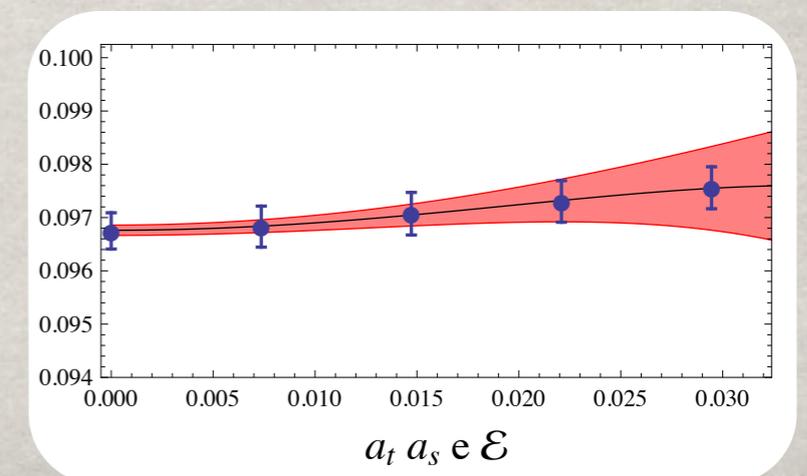
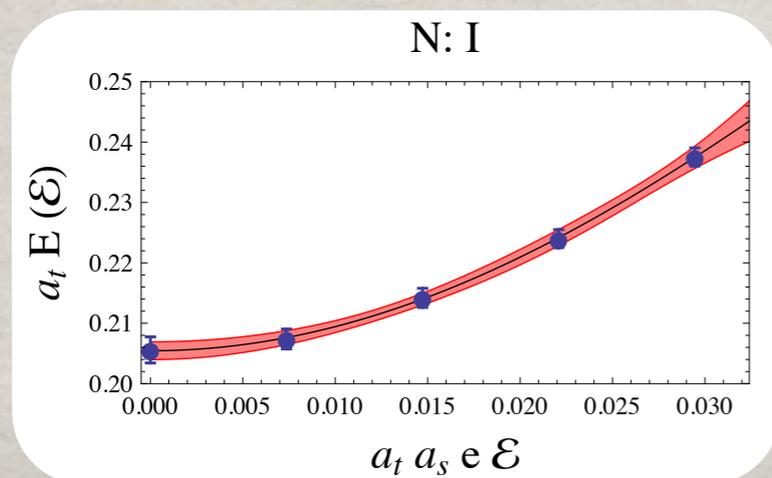
POLARIZABILITY TEASER

$$E = M_N + 4\pi\alpha_E \mathcal{E}^2 + \dots$$

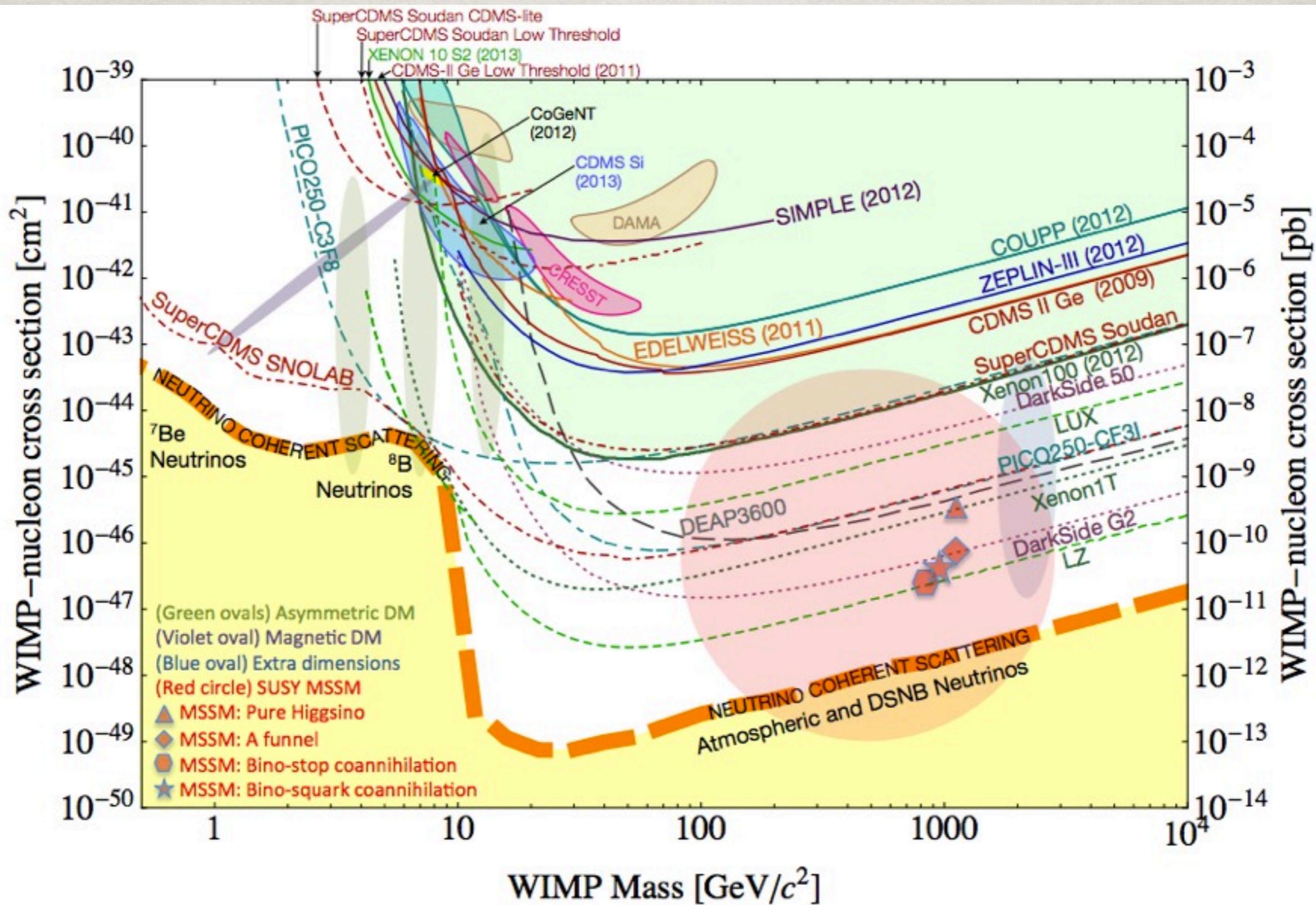


Neutron (Detmold, Tiburzi, Walker-Loud, 2010)

Neutral Kaon (Detmold, Tiburzi, Walker-Loud, 2009)

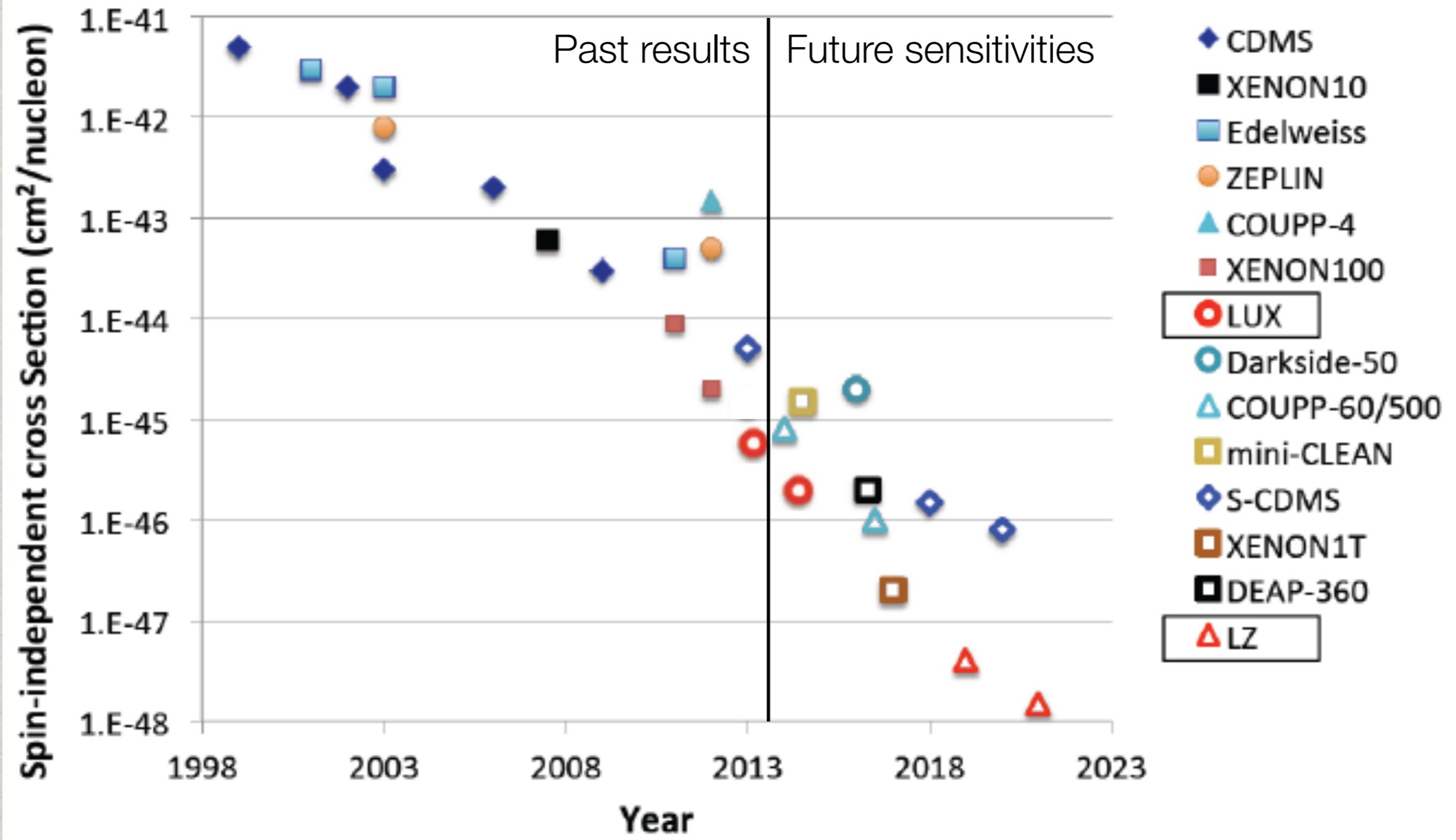


Backup



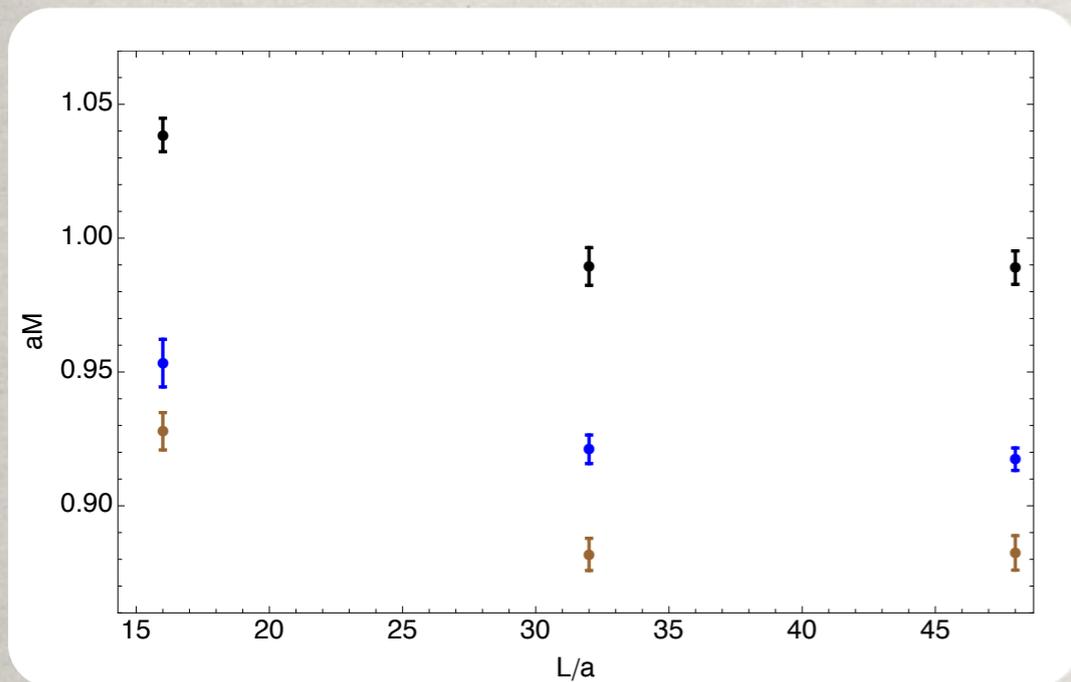
(Blair Edwards Presentation- Lattice Meets Experiment 2013)

Spin-Independent cross section limits for 50 GeV WIMP versus time, including future projections

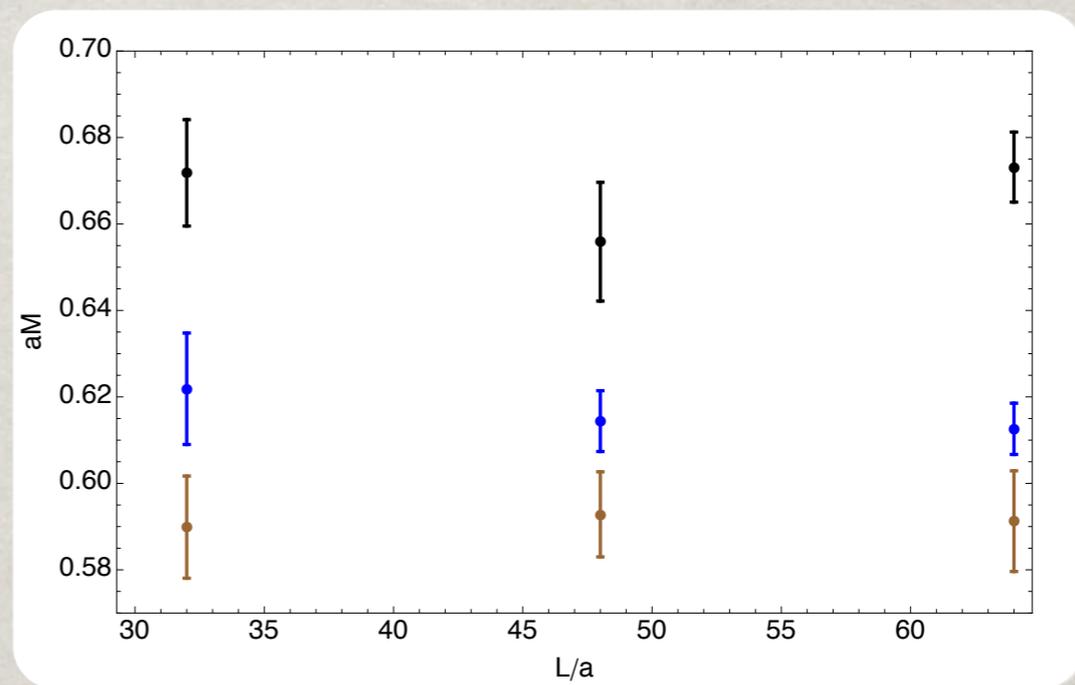
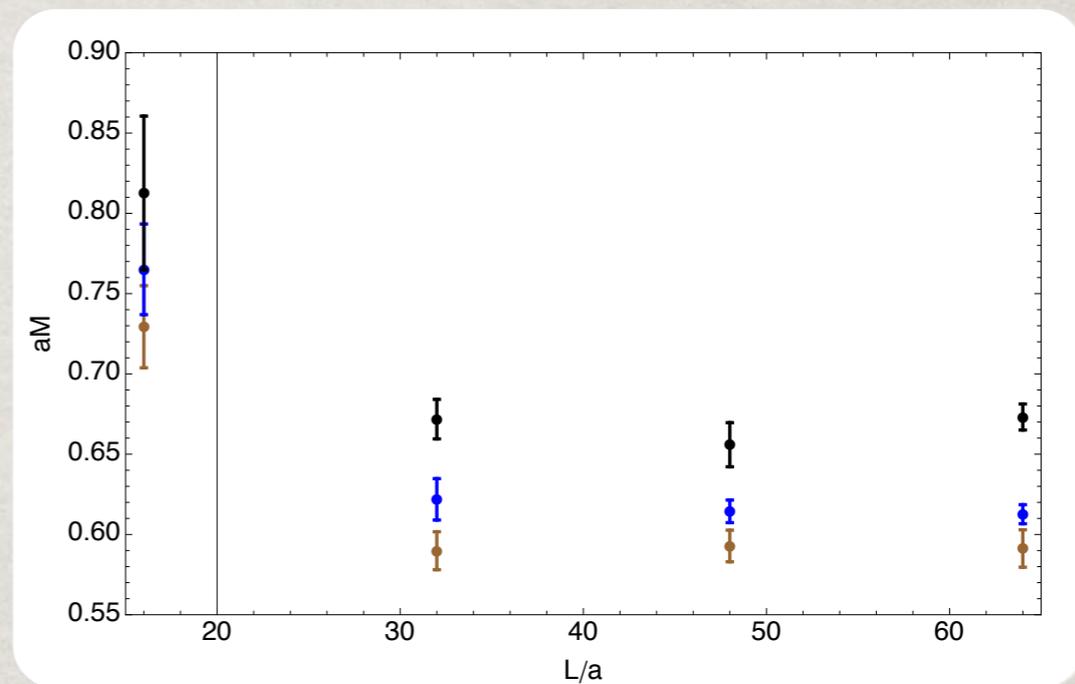


(Blair Edwards Presentation- Lattice Meets Experiment 2013)

VOLUME EFFECTS

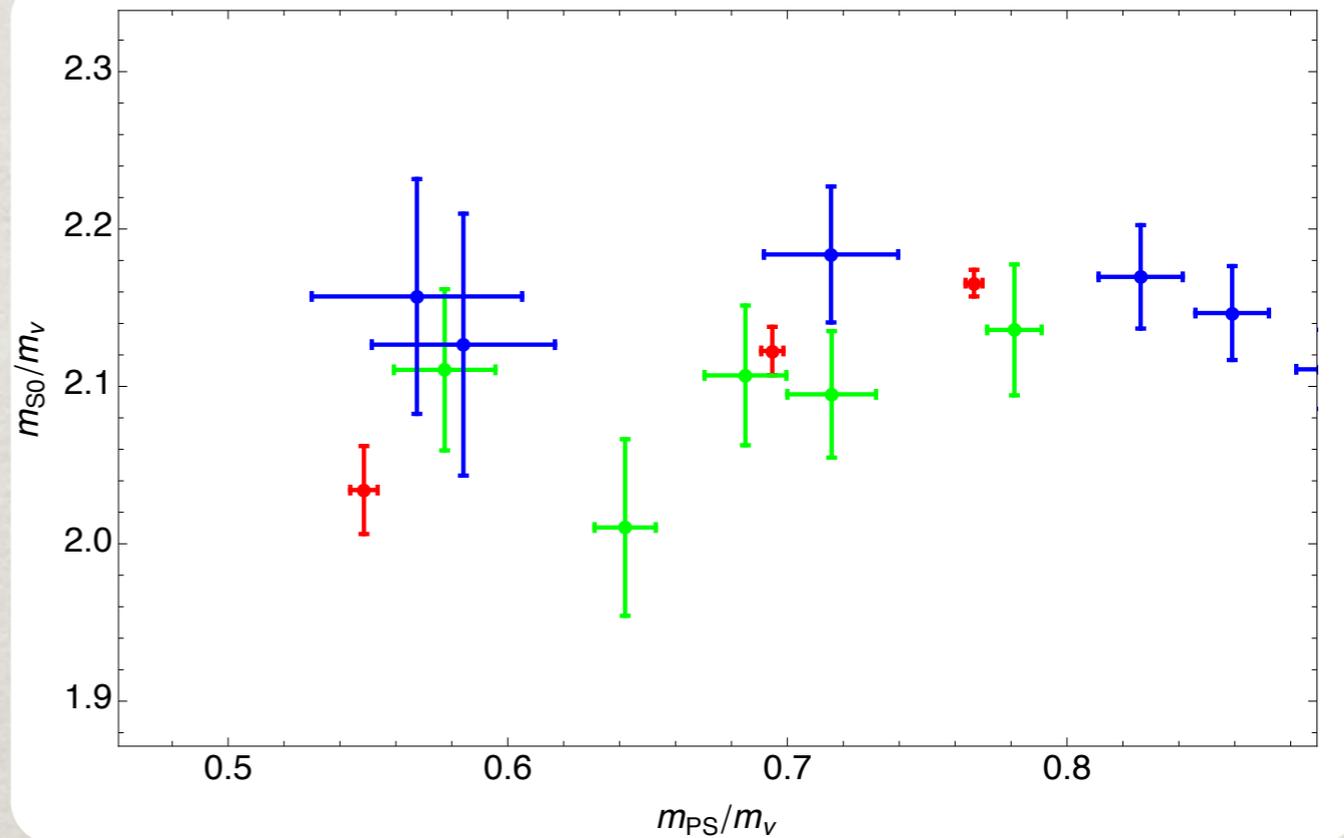
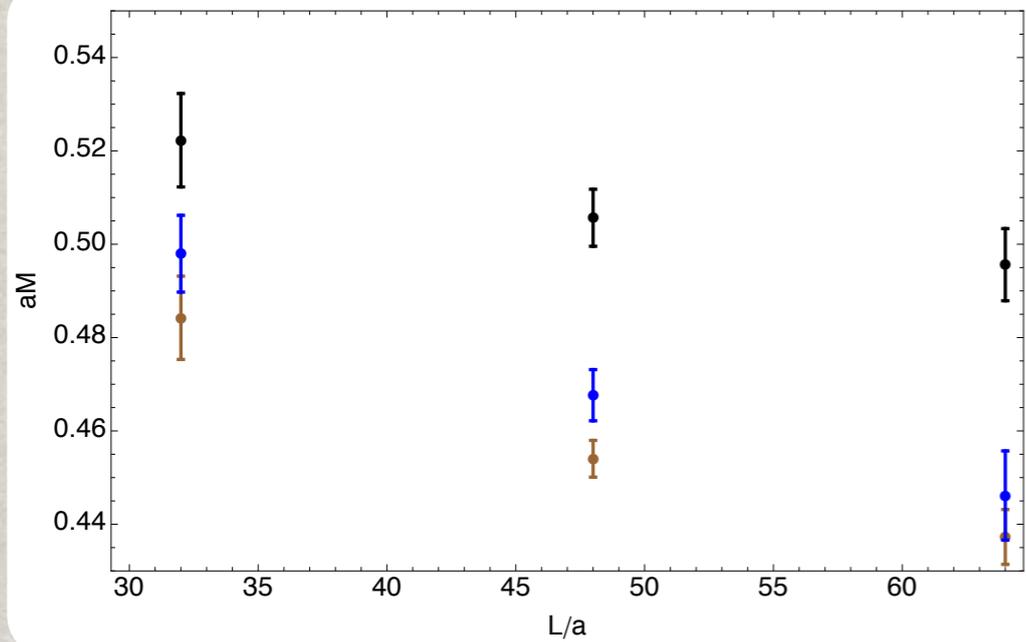
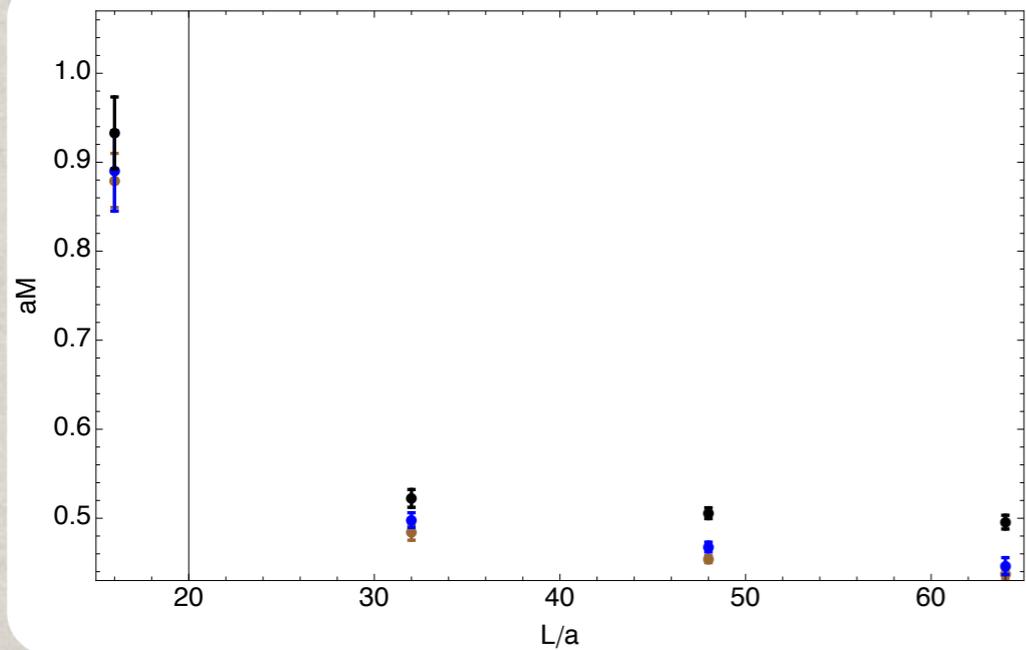


$$\beta = 11.028$$



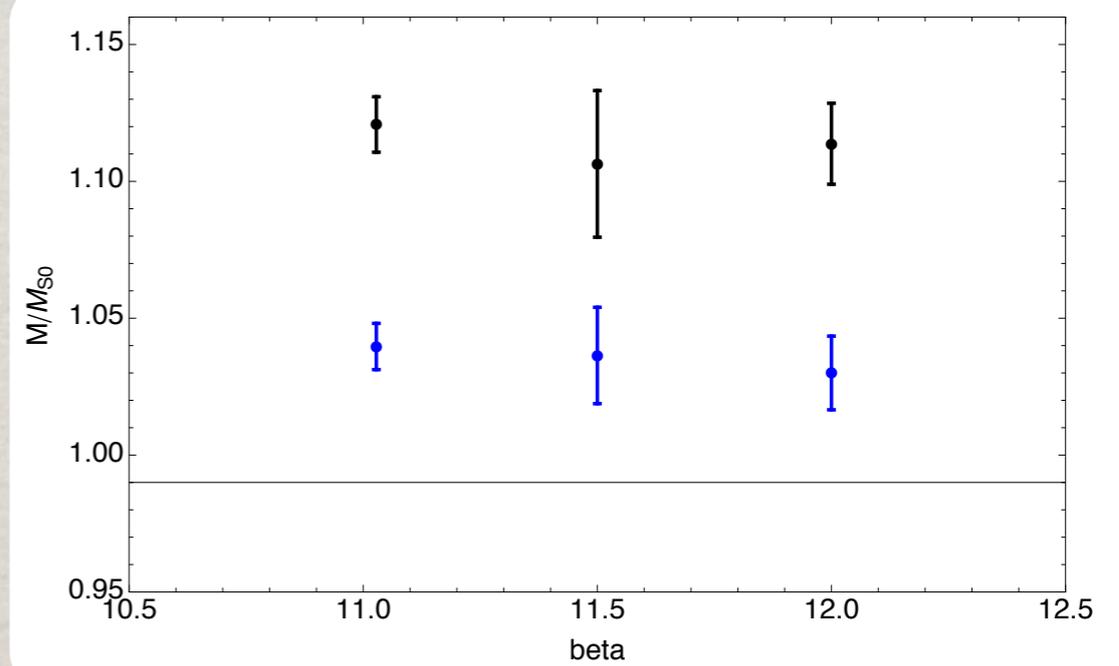
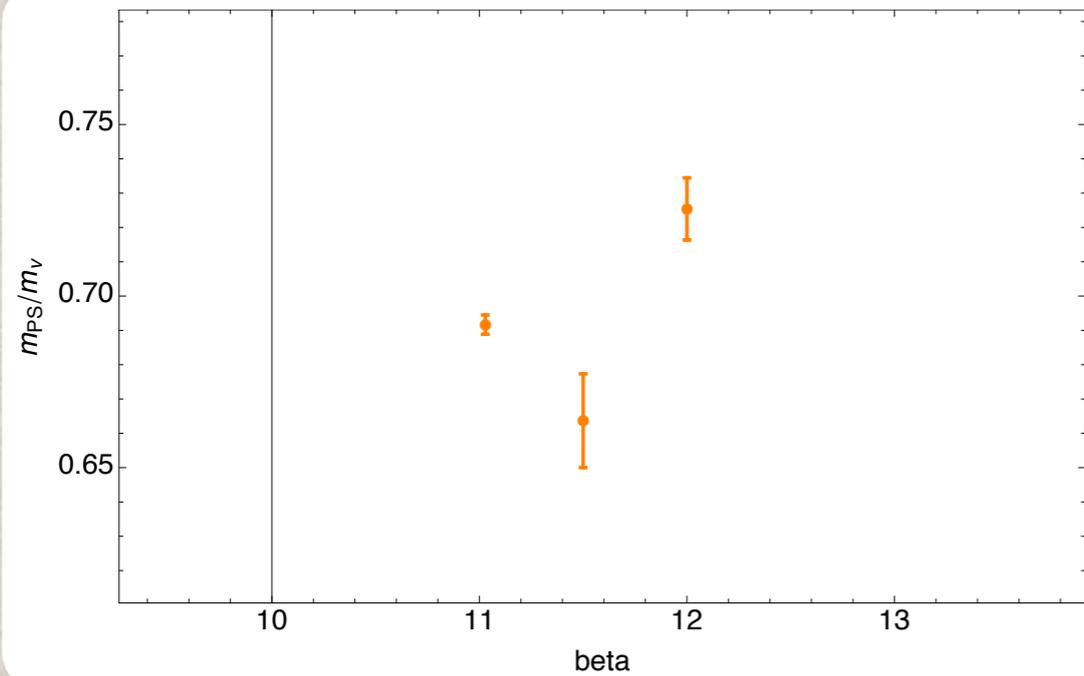
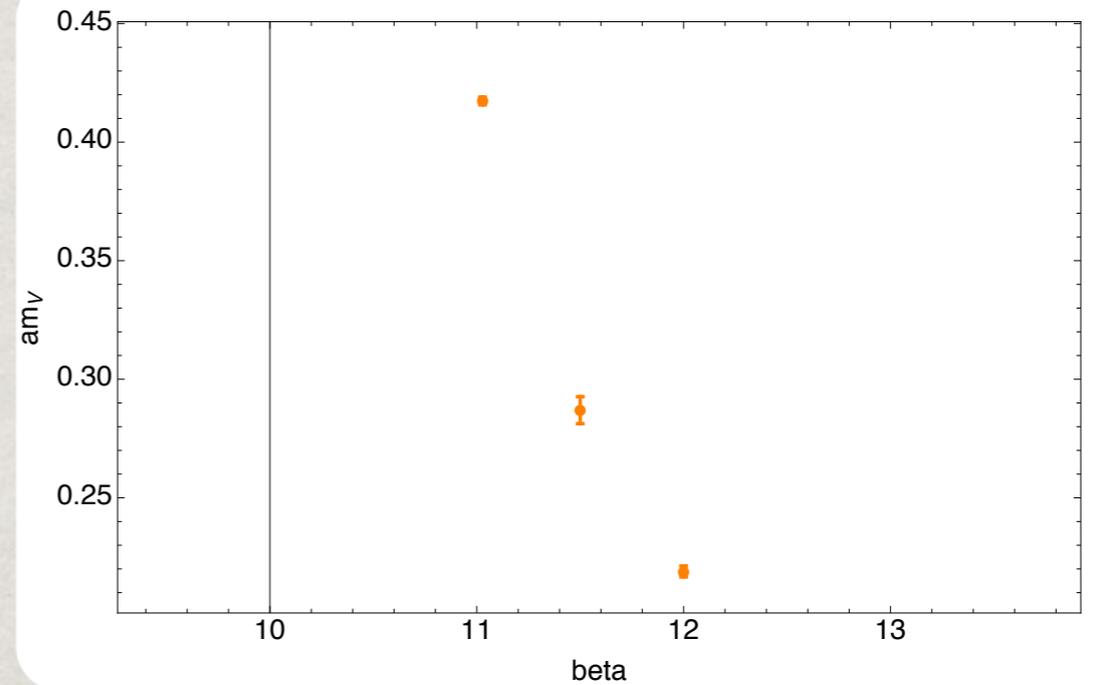
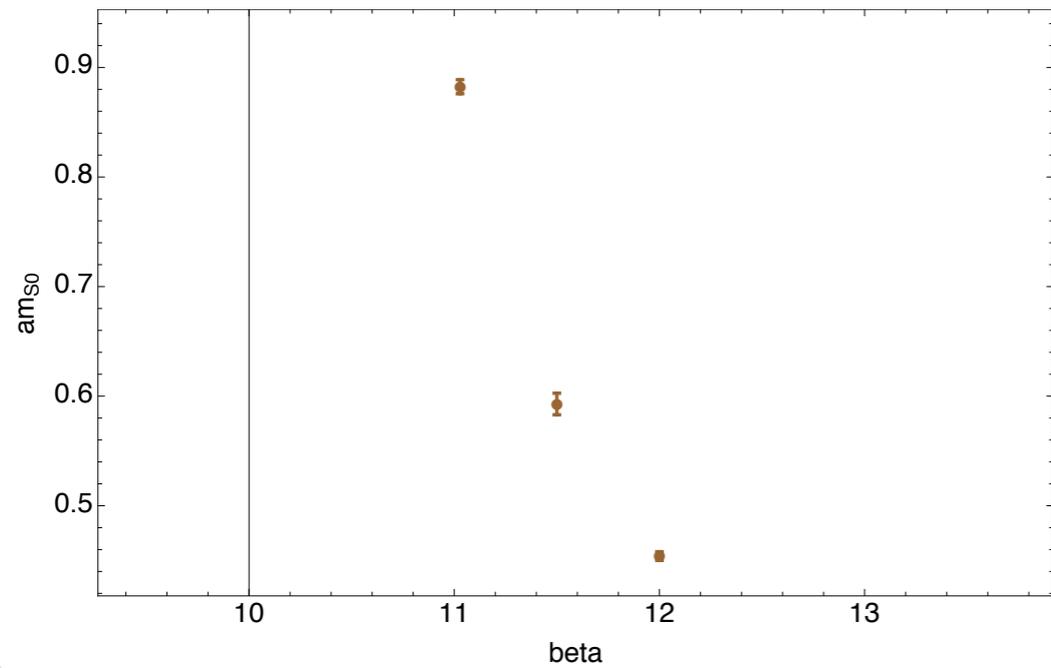
$$\beta = 11.5$$

VOLUME EFFECTS



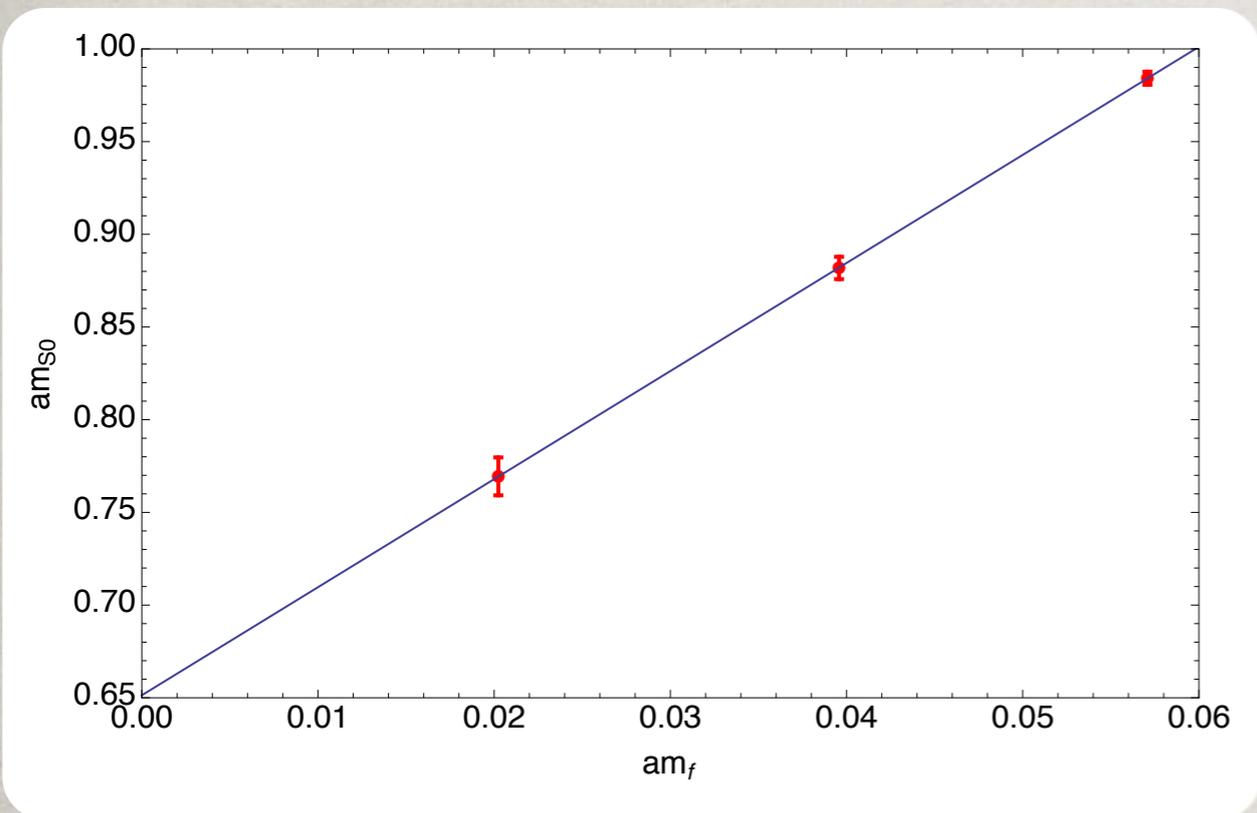
$$\beta = 12.0$$

LATTICE SPACING EFFECTS

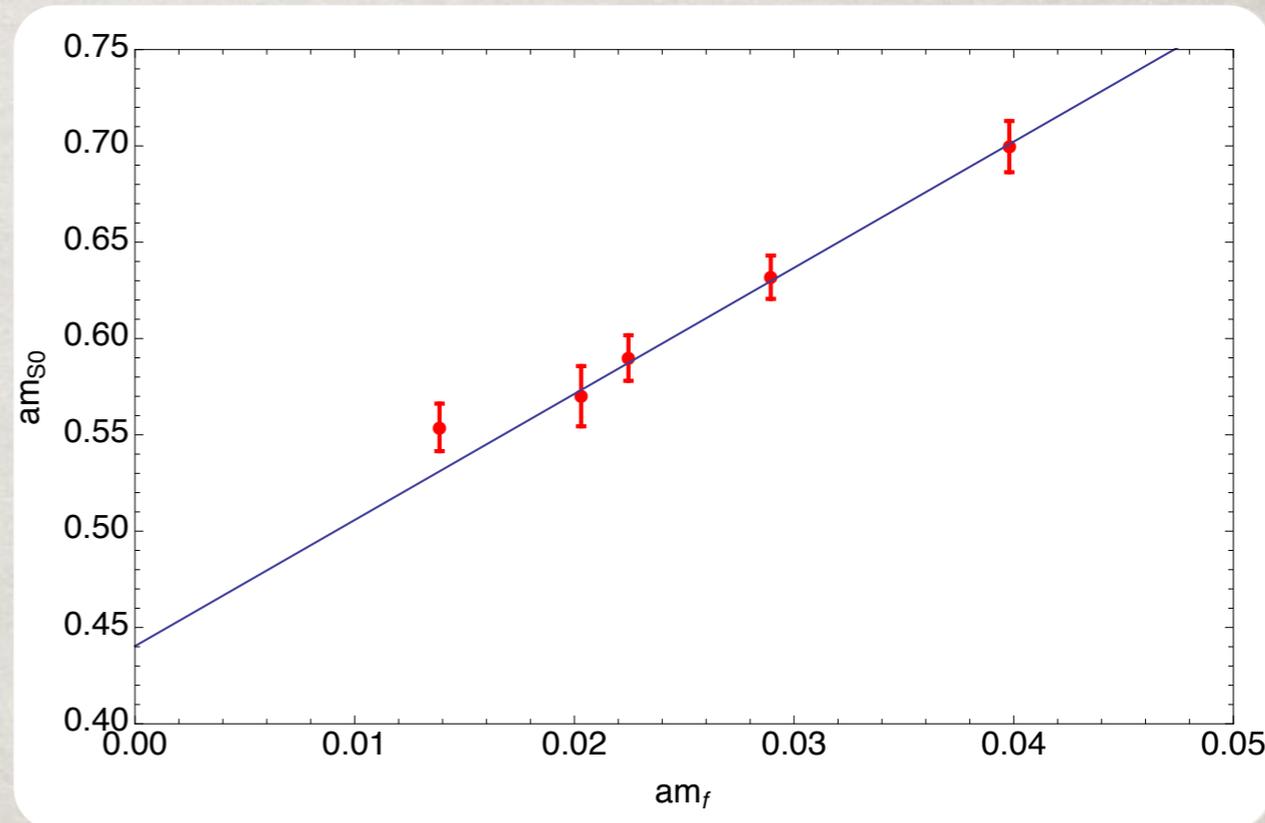


BARYON MASS DERIVATIVE

Coarse lattice spacing



Intermediate lattice spacing

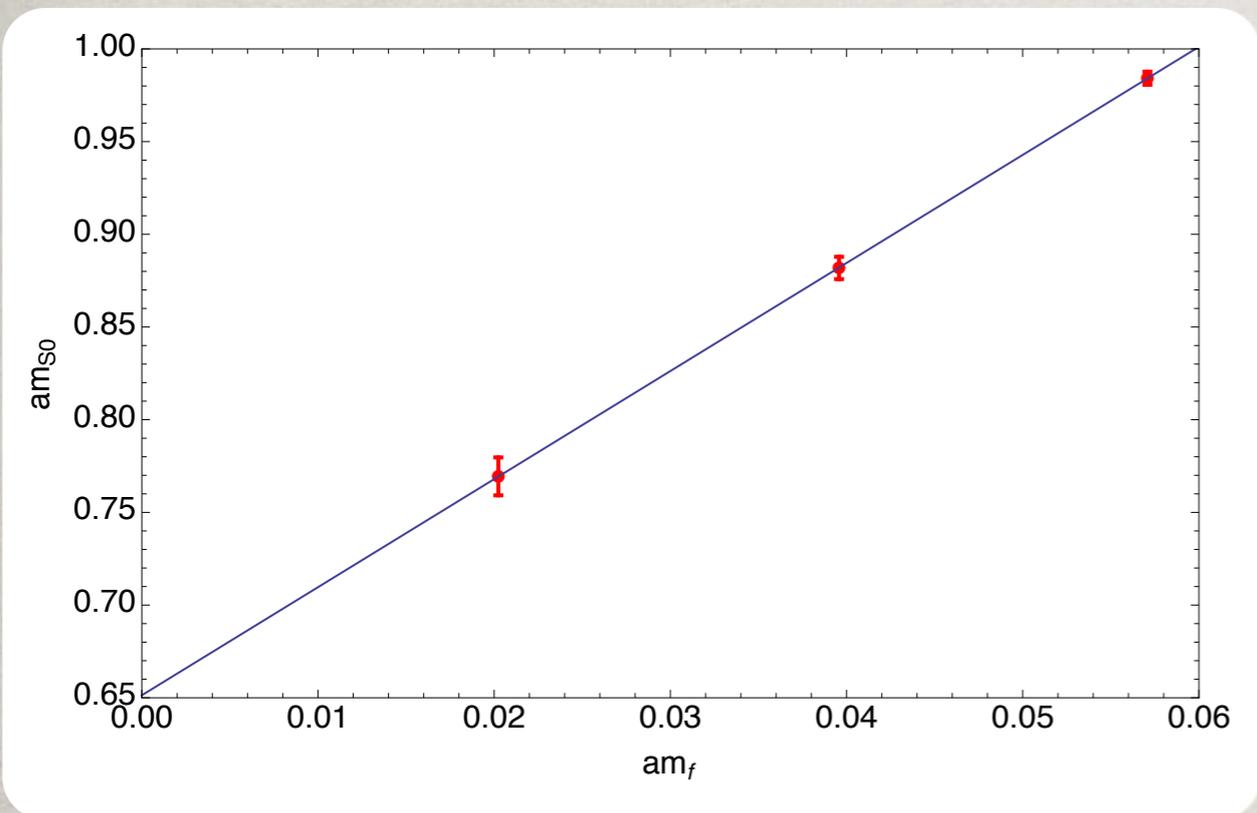


$$\frac{\partial m_B}{\partial m_f} = 5.83(30)$$

$$\frac{\partial m_B}{\partial m_f} = 6.55(91)$$

BARYON MASS DERIVATIVE

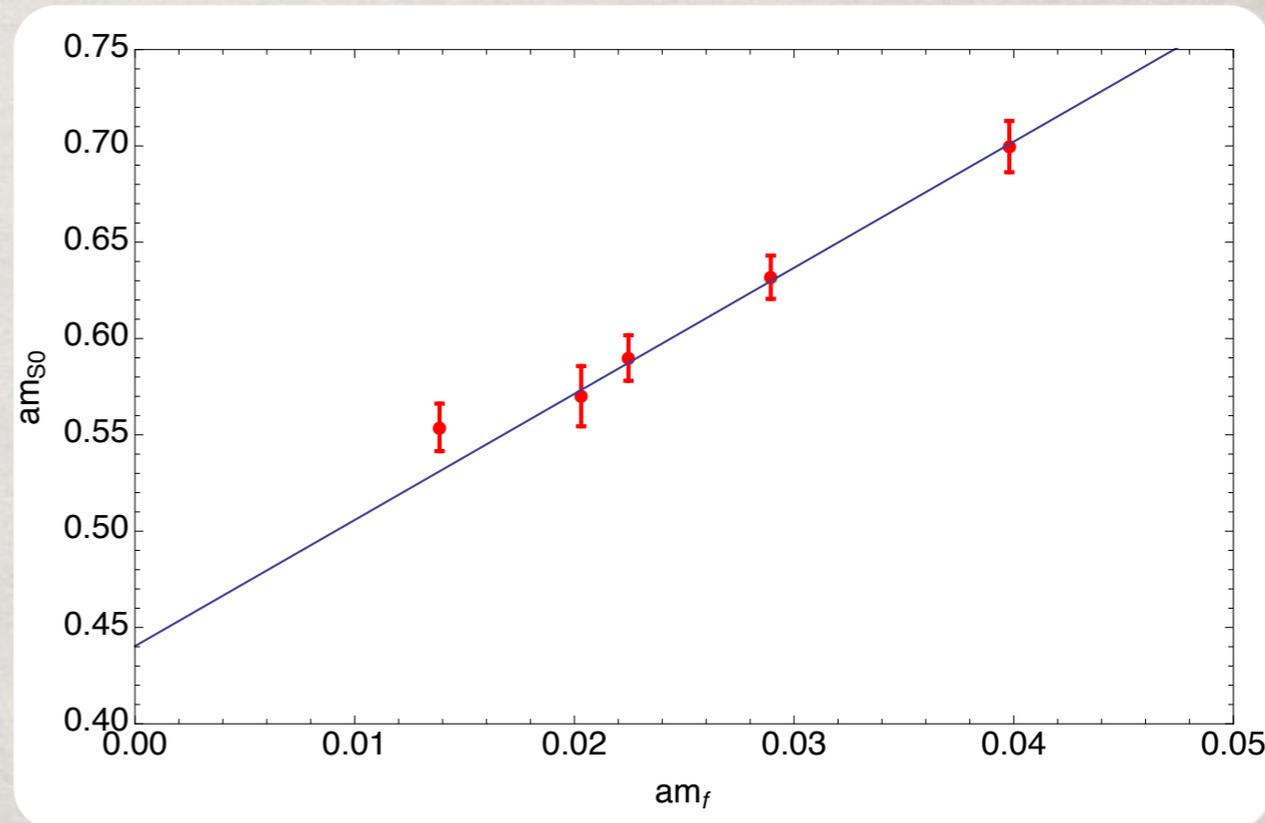
Coarse lattice spacing



$$\frac{m_{PS}}{m_V} = 0.695(4)$$

$$\frac{m_f}{m_B} \frac{\partial m_B}{\partial m_f} = 0.261(14)$$

Intermediate lattice spacing

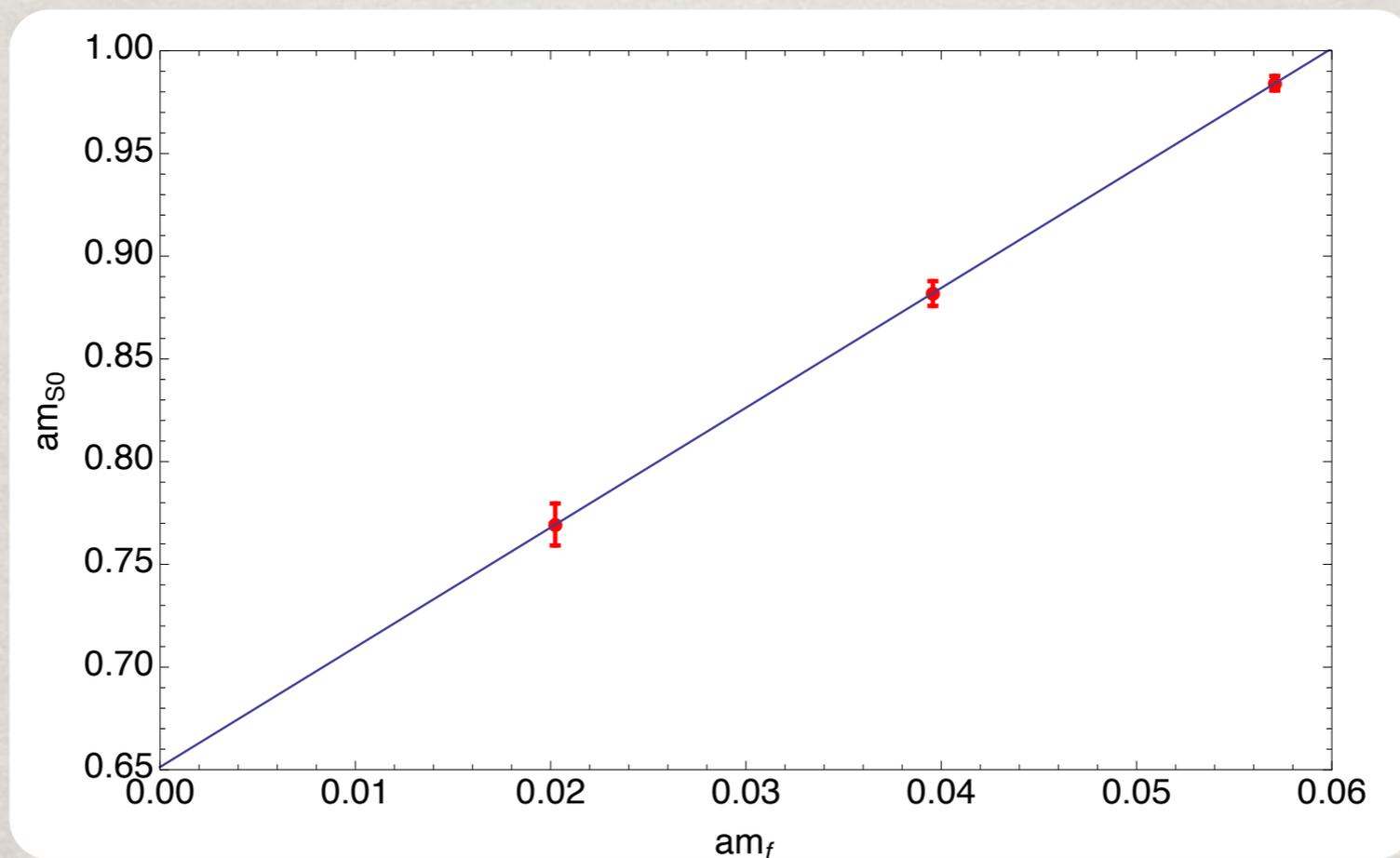


$$\frac{m_{PS}}{m_V} = 0.685(14)$$

$$\frac{m_f}{m_B} \frac{\partial m_B}{\partial m_f} = 0.249(35)$$

BARYON MASS DERIVATIVE

Coarse lattice spacing



$$\frac{m_{PS}}{m_V} \approx 0.55 \quad \rightarrow \quad 0.153 \lesssim \frac{m_f}{m_B} \frac{\partial m_B}{\partial m_f} \lesssim 0.338 \quad \leftarrow \quad \frac{m_{PS}}{m_V} \approx 0.77$$

VEC. MASS SUPPRESSION?

✻ Mass Matrix: $\psi_L \equiv \begin{pmatrix} F_1 \\ F_2^\dagger \end{pmatrix}, \psi_R \equiv \begin{pmatrix} F_3 \\ F_4^\dagger \end{pmatrix}$

$$\langle H \rangle = \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} \quad \rightarrow \quad \mathcal{L}_M = (\bar{\psi}_L \bar{\psi}_R) \begin{pmatrix} m_{12} & yv \\ yv & m_{34} \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

VEC. MASS SUPPRESSION?

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$$m_{\pm} = \frac{1}{2} \left(m_{12} + m_{34} \pm \sqrt{4y^2v^2 + (m_{34} - m_{12})^2} \right) \quad \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} \sin \theta & \cos \theta \\ \cos \theta & -\sin \theta \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

Lattice Mass Lattice Fermion

VEC. MASS SUPPRESSION?

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Lattice Mass Lattice Fermion

$$(\bar{\psi}_L \bar{\psi}_R) \begin{pmatrix} 0 & yh \\ yh & 0 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad \longrightarrow \quad yh (\bar{\psi}_- \bar{\psi}_+) \begin{pmatrix} 2 \cos \theta \sin \theta & \cos^2 \theta - \sin^2 \theta \\ \cos^2 \theta - \sin^2 \theta & -2 \cos \theta \sin \theta \end{pmatrix} \begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix}$$

VEC. MASS SUPPRESSION?

☀ Mass Matrix: $\psi_L \equiv \begin{pmatrix} F_1 \\ F_2^\dagger \end{pmatrix}, \psi_R \equiv \begin{pmatrix} F_3 \\ F_4^\dagger \end{pmatrix}$

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Lattice Mass Lattice Fermion

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$$y_f = 2y \cos \theta \sin \theta \quad y_f \rightarrow \begin{cases} y & 2yv \gg (m_{34} - m_{12}) \\ \frac{2y^2v}{(m_{34} - m_{12})} & (m_{34} - m_{12}) \gg 2yv \end{cases}$$

VEC. MASS SUPPRESSION?

☀ Mass Matrix: $\psi_L \equiv \begin{pmatrix} F_1 \\ F_2^\dagger \end{pmatrix}, \psi_R \equiv \begin{pmatrix} F_3 \\ F_4^\dagger \end{pmatrix}$

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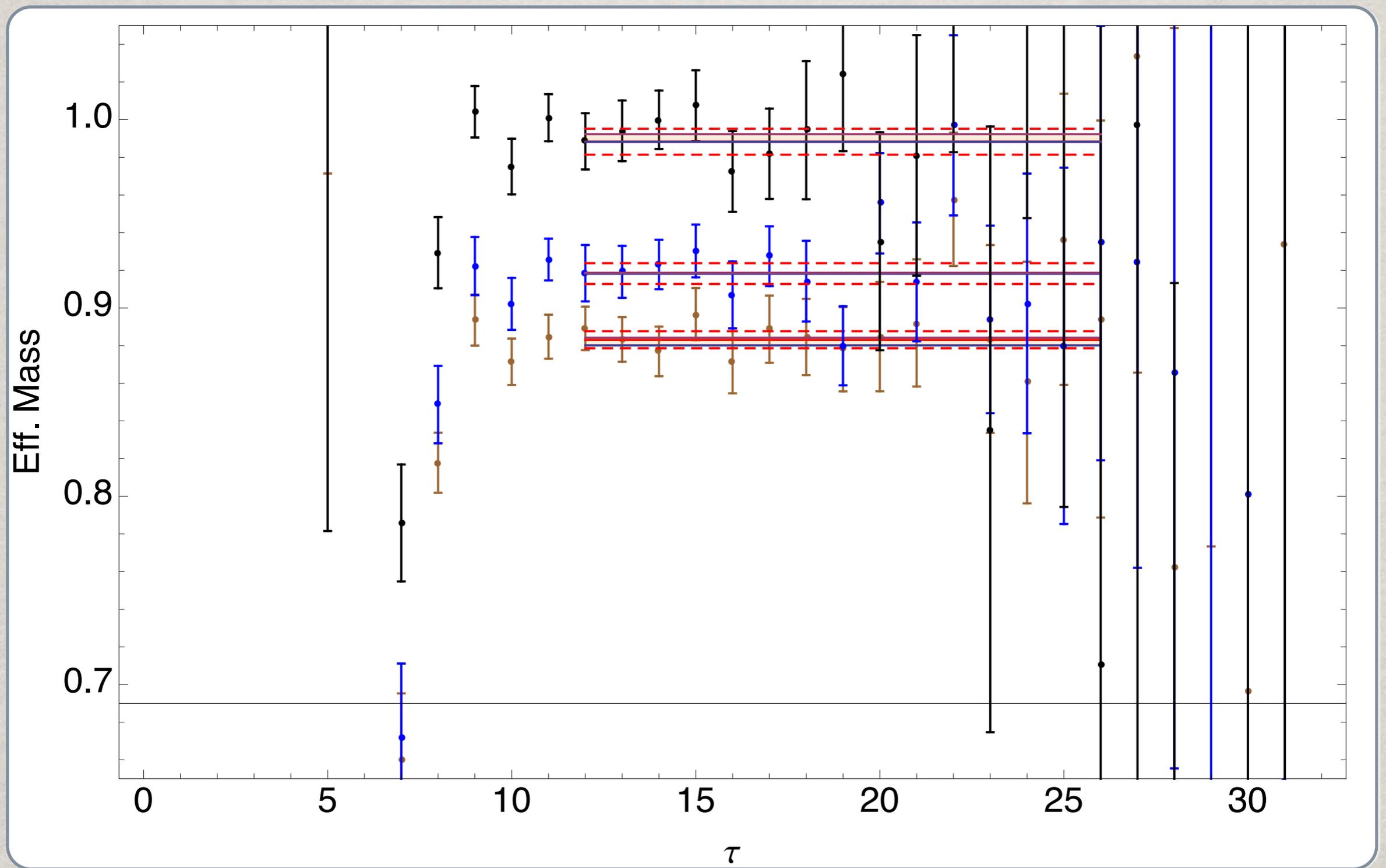
$$m_{\pm} = \frac{1}{2} \left(m_{12} + m_{34} \pm \sqrt{4y^2v^2 + (m_{34} - m_{12})^2} \right) \quad \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} \sin \theta & \cos \theta \\ \cos \theta & -\sin \theta \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

Lattice Mass Lattice Fermion

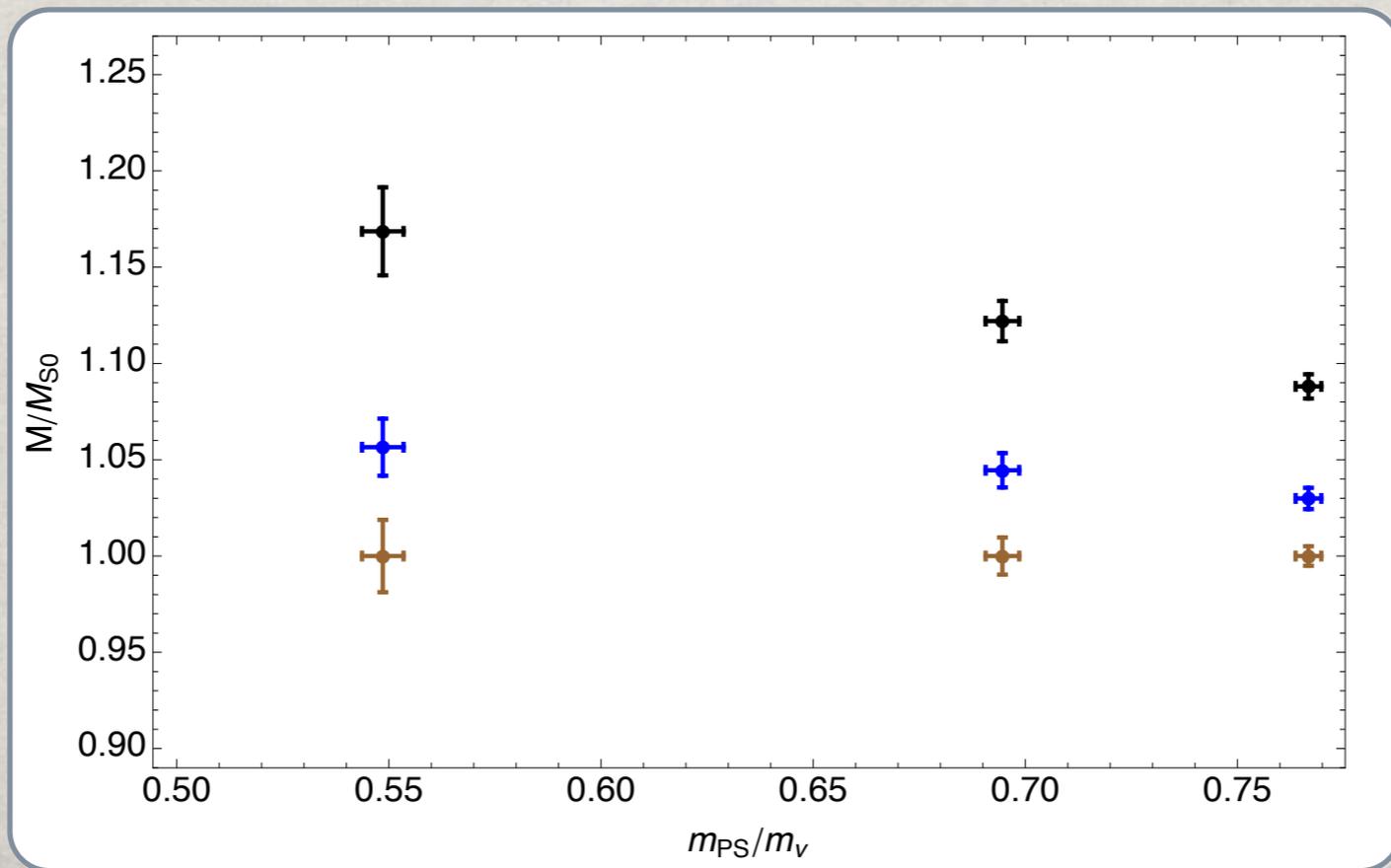
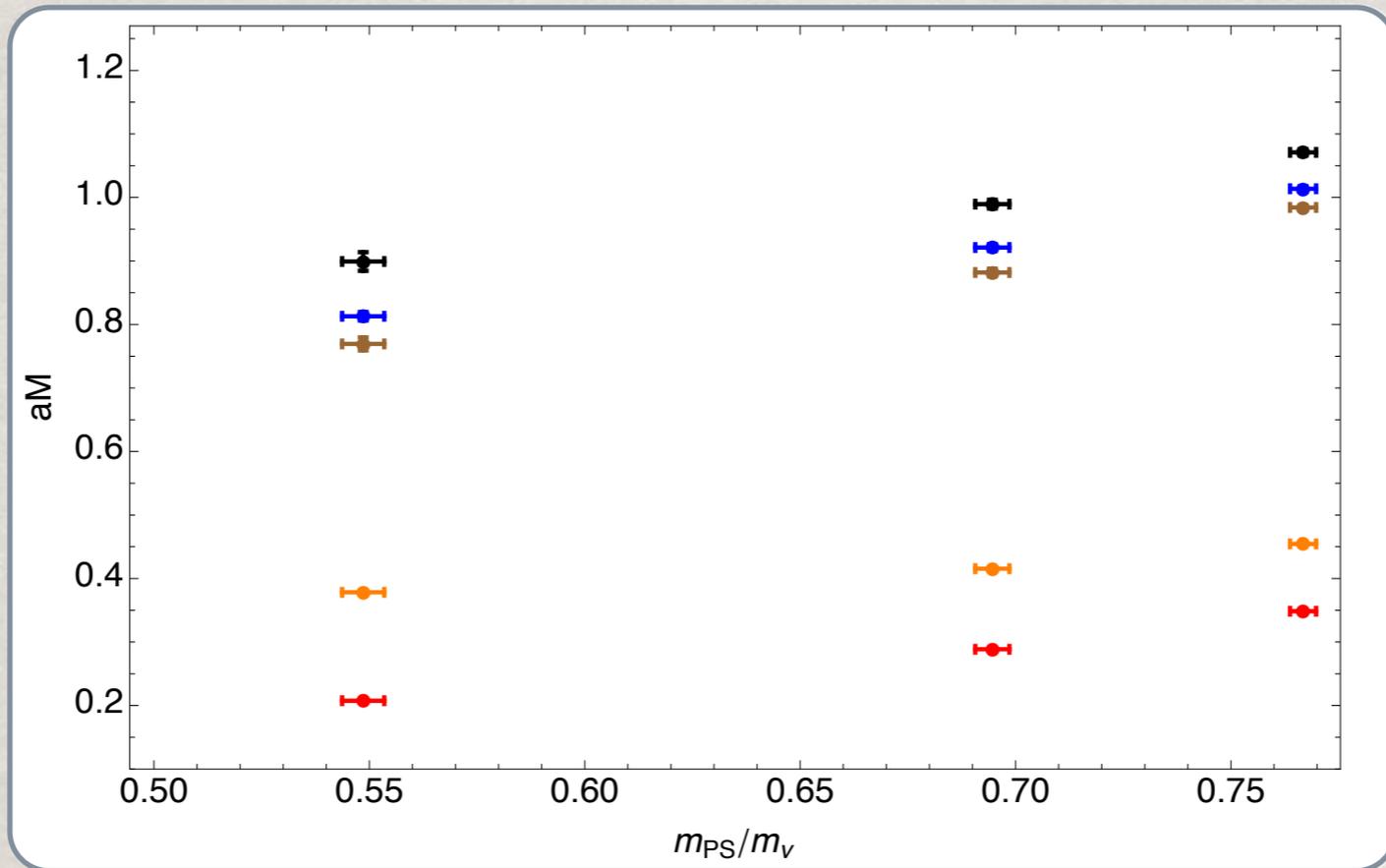
$$(\bar{\psi}_L \bar{\psi}_R) \begin{pmatrix} 0 & yh \\ yh & 0 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad \longrightarrow \quad yh (\bar{\psi}_- \bar{\psi}_+) \begin{pmatrix} 2 \cos \theta \sin \theta & \cos^2 \theta - \sin^2 \theta \\ \cos^2 \theta - \sin^2 \theta & -2 \cos \theta \sin \theta \end{pmatrix} \begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix}$$

$$y_f = 2y \cos \theta \sin \theta \quad \text{Suppression} \quad y_f \rightarrow \begin{cases} y & 2yv \gg (m_{34} - m_{12}) \\ \frac{2y^2v}{(m_{34} - m_{12})} & (m_{34} - m_{12}) \gg 2yv \end{cases}$$

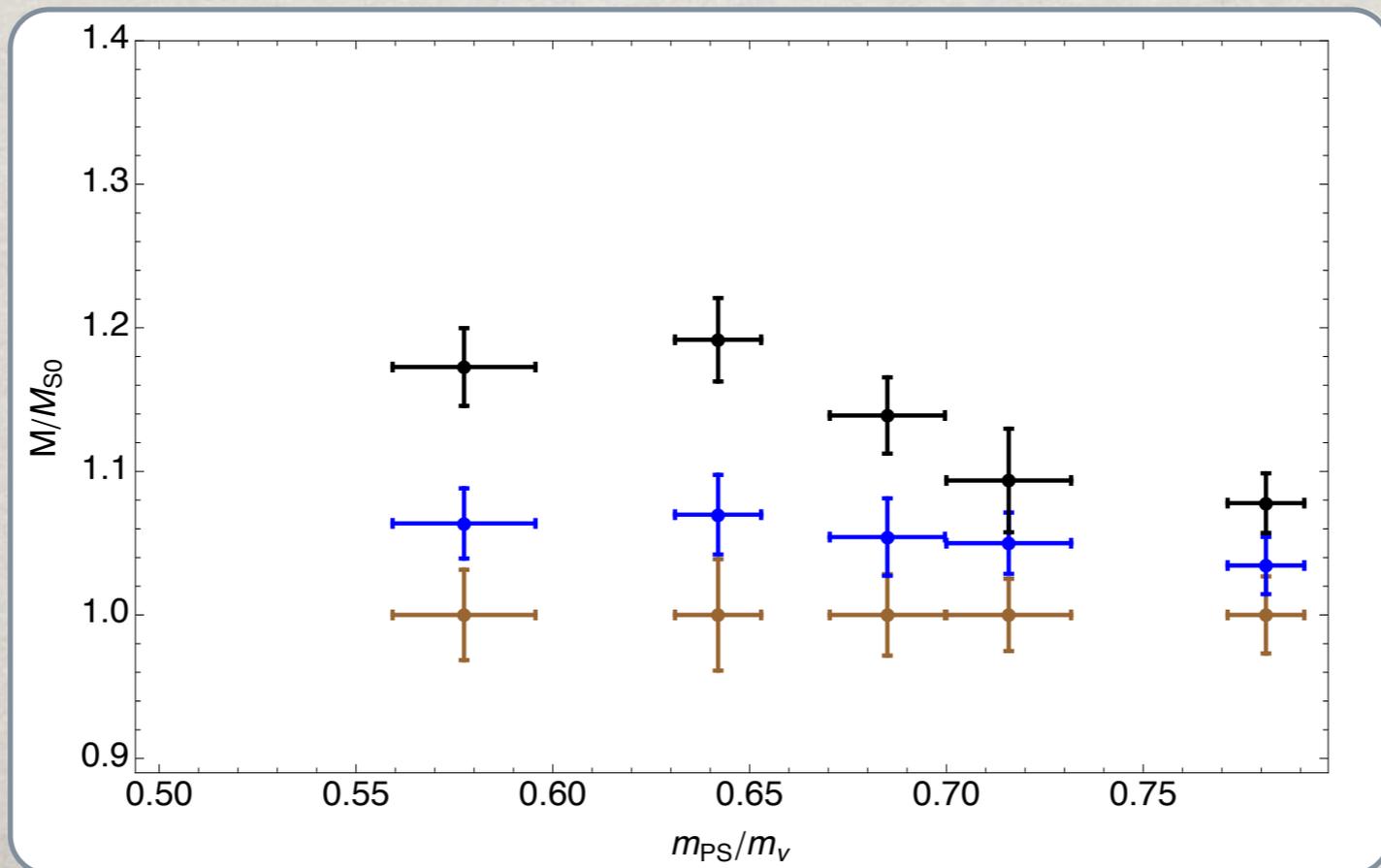
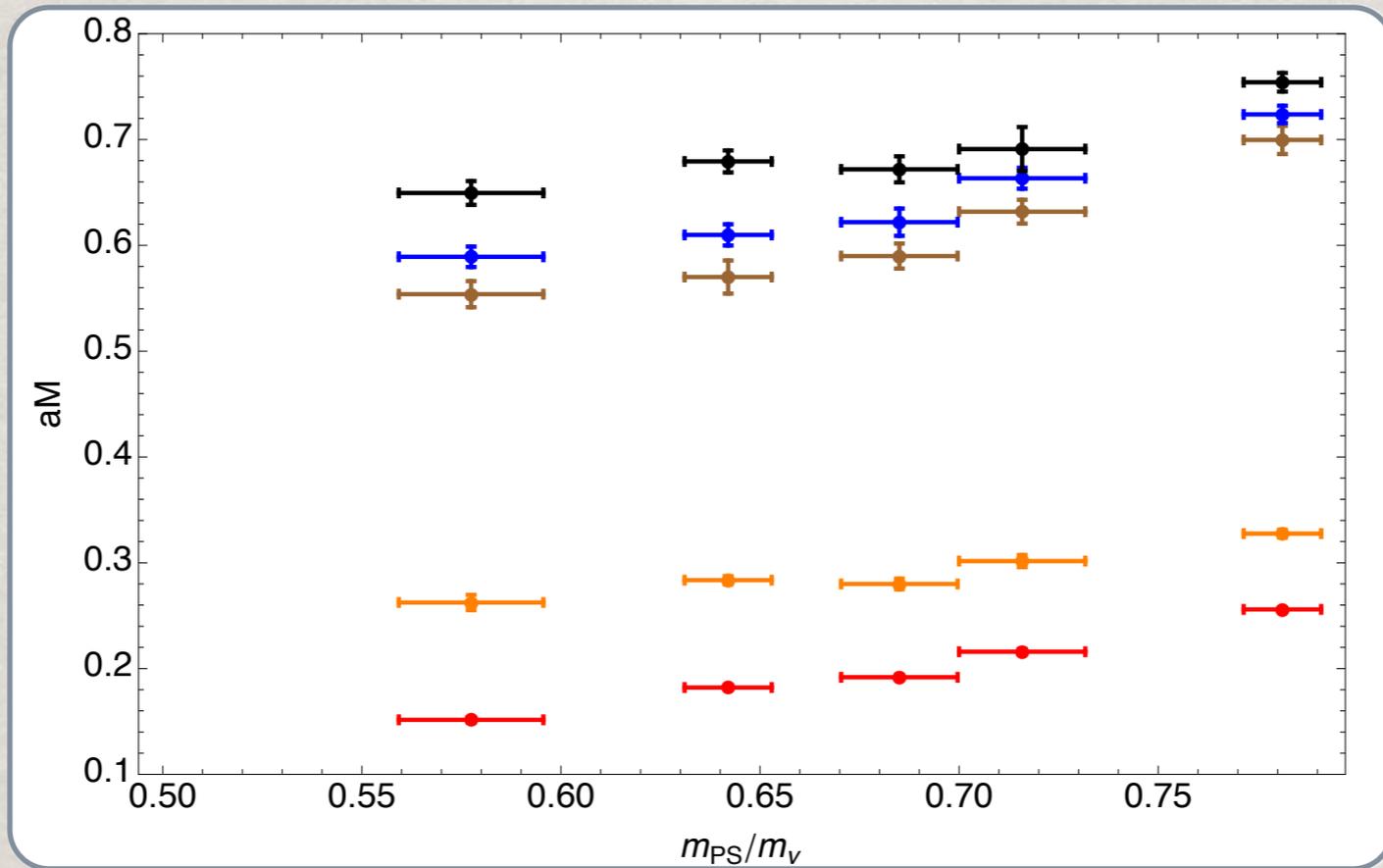
EFFECTIVE MASS EXAMPLE



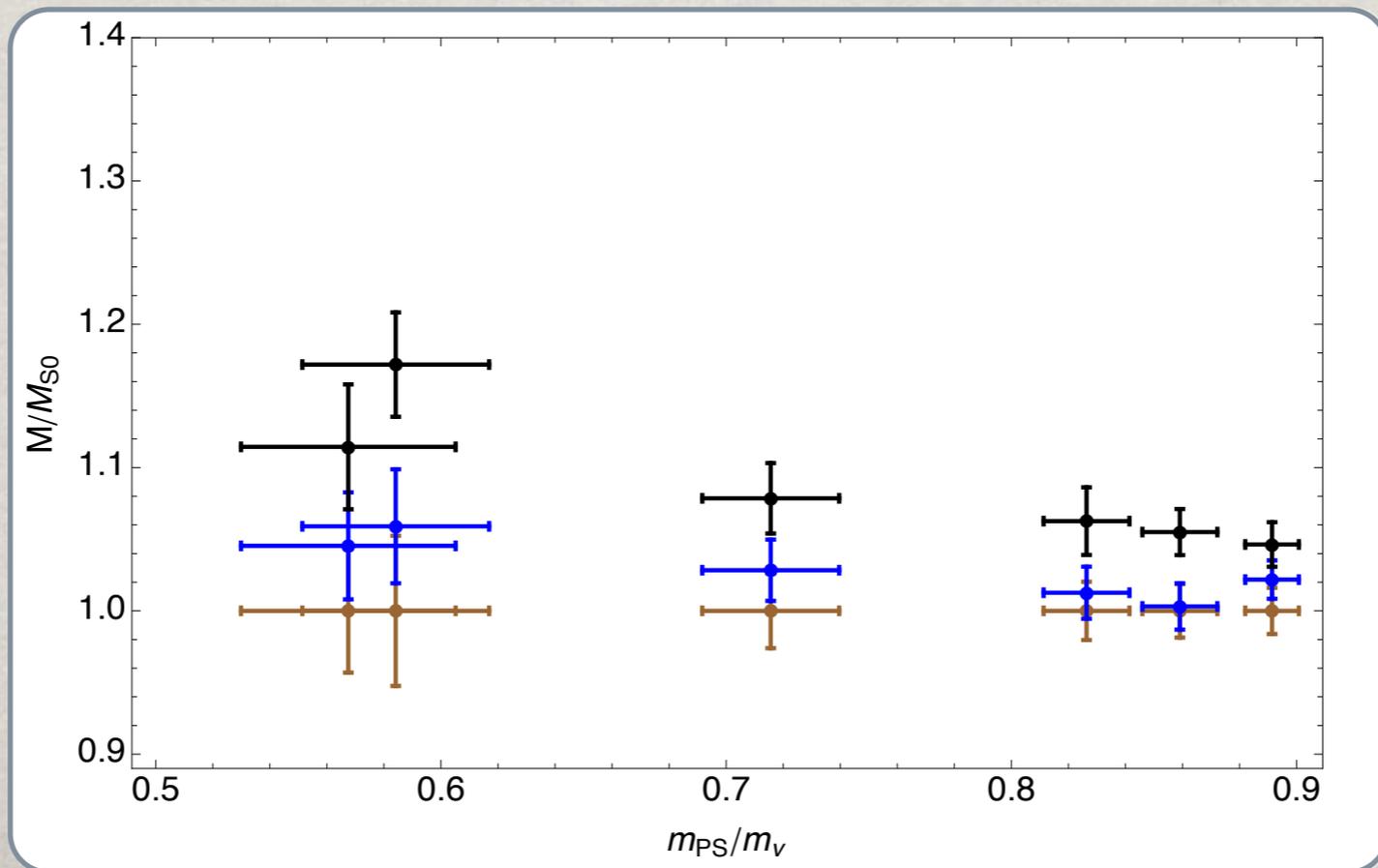
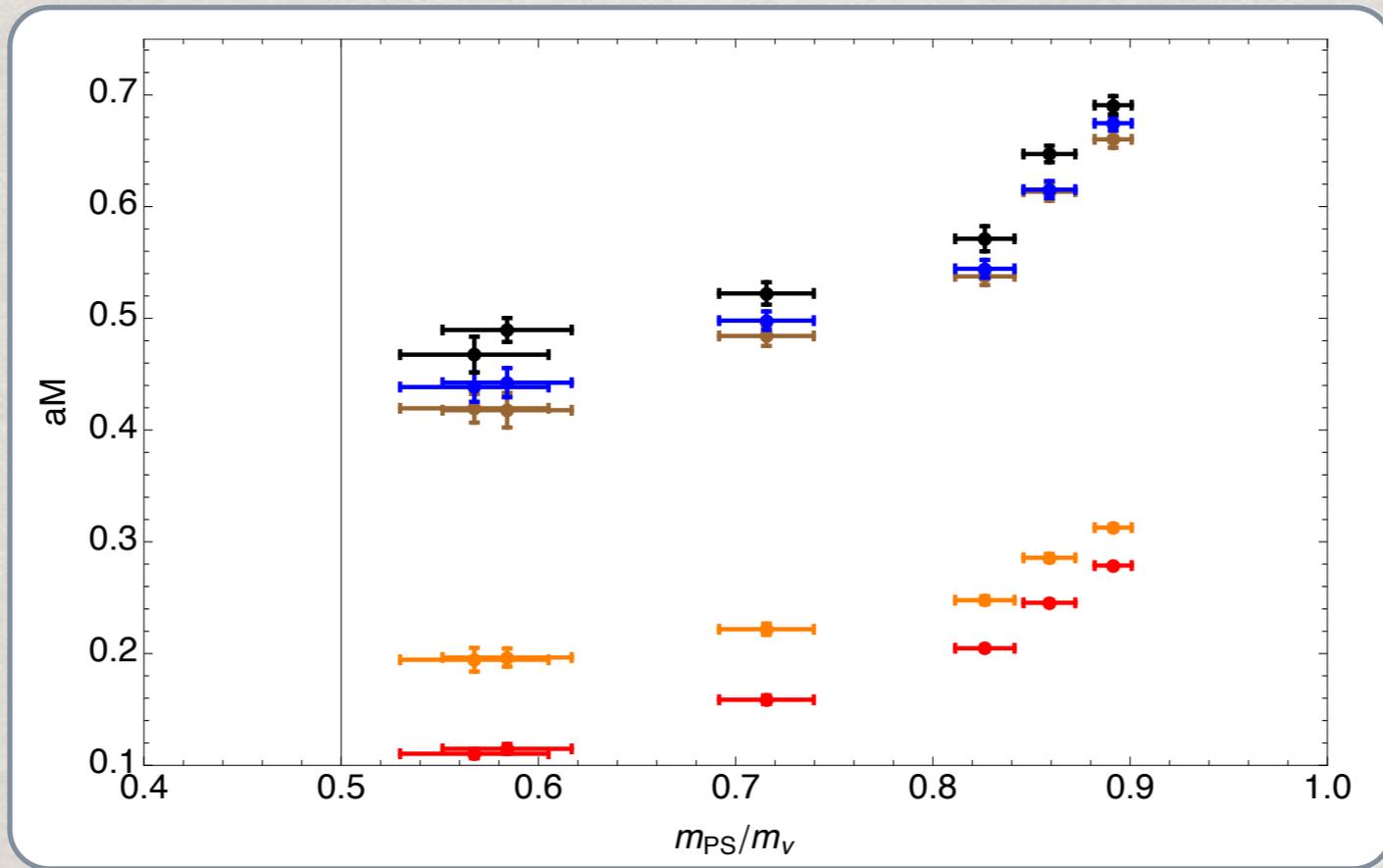
COARSE LATTICE SPACING



INTERMED. LATTICE SPACING



INTERMED. LATTICE SPACING



TIGHT CONSTRAINTS?

- ✿ Assume a Dirac particle with net Z-boson charge

$$\sigma_{SI} \approx \frac{2}{\pi} G_F^2 m_N^2 \frac{\bar{N}^2}{A^2} \approx \frac{\bar{N}^2}{A^2} (3 \times 10^{-38} \text{ cm}^2) \quad \frac{\bar{N}^2}{A^2} \sim \frac{1}{4}$$

Current spin-independent bounds: $\sigma \lesssim 10^{-45} \text{ cm}^2$

Excludes particles of this kind to masses greater than thousands of TeV

Neutralinos avoid this:

Majorana



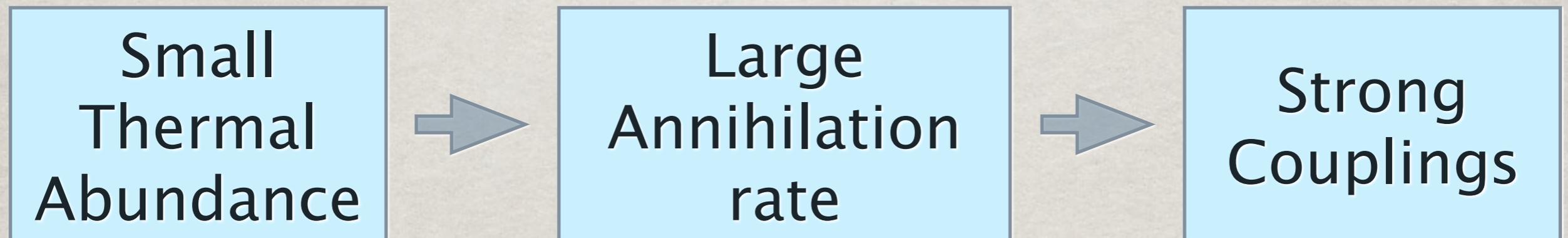
Spin-Dependent

This will plague composites with odd numbers of EW doublets!

THERMAL VS. ASYMMETRIC

However:

Asymmetric relic density
suggests negligible thermal abundance



Tricky to achieve for perturbative, elementary DM

Strongly-coupled composite theories most interesting...

...this is where the lattice can play significant role!

PARTICULAR MODEL

☀ Four Dirac Flavors with vector-like masses

Field	$SU(N)_D$	$(SU(2)_L, Y)$	Q
$F_1 = \begin{pmatrix} F_1^u \\ F_1^d \end{pmatrix}$	\mathbf{N}	$(\mathbf{2}, 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
$F_2 = \begin{pmatrix} F_2^u \\ F_2^d \end{pmatrix}$	$\bar{\mathbf{N}}$	$(\mathbf{2}, 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
F_3^u	\mathbf{N}	$(\mathbf{1}, +1/2)$	$+1/2$
F_3^d	\mathbf{N}	$(\mathbf{1}, -1/2)$	$-1/2$
F_4^u	$\bar{\mathbf{N}}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_4^d	$\bar{\mathbf{N}}$	$(\mathbf{1}, -1/2)$	$-1/2$

Kinetic:

$$\begin{aligned} \mathcal{L}_D = & iF_1^\dagger \bar{\sigma}^\mu \nabla_{\mu,L} F_1 + iF_2^\dagger \bar{\sigma}^\mu \nabla_{\mu,L}^* F_2 + iF_3^{u\dagger} \bar{\sigma}^\mu \nabla_{\mu,R} F_3^{u\dagger} + iF_3^{d\dagger} \bar{\sigma}^\mu \nabla_{\mu,R} F_3^{d\dagger} \\ & + iF_4^{u\dagger} \bar{\sigma}^\mu \nabla_{\mu,R}^* F_4^{u\dagger} + iF_4^{d\dagger} \bar{\sigma}^\mu \nabla_{\mu,R}^* F_4^{d\dagger} \end{aligned}$$

$$\nabla_L^\mu = \partial^\mu + ig A^{a,\mu} (\tau_L^a / 2)$$

$$(\nabla_L^\mu)^* = \partial^\mu - ig A^{a,\mu} (\tau_L^a / 2)$$

$$\nabla_R^\mu = \partial^\mu + ig' B^\mu (\tau_R^3 / 2)$$

$$(\nabla_R^\mu)^* = \partial^\mu - ig' B^\mu (\tau_R^3 / 2)$$



Left/Right
Different

PARTICULAR MODEL

☀ Four Dirac Flavors with vector-like masses

Field	SU(N) _D	(SU(2) _L , Y)	Q
$F_1 = \begin{pmatrix} F_1^u \\ F_1^d \end{pmatrix}$	N	(2 , 0)	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
$F_2 = \begin{pmatrix} F_2^u \\ F_2^d \end{pmatrix}$	$\bar{\mathbf{N}}$	(2 , 0)	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
F_3^u	N	(1 , +1/2)	+1/2
F_3^d	N	(1 , -1/2)	-1/2
F_4^u	$\bar{\mathbf{N}}$	(1 , +1/2)	+1/2
F_4^d	$\bar{\mathbf{N}}$	(1 , -1/2)	-1/2

Masses:

Chiral

$$\mathcal{L}_{\mathcal{M}} = -y_{14}^u \epsilon_{ij} F_1^i H^j F_4^d - y_{14}^d \delta_{ij} F_1^i (H^\dagger)^j F_4^u + y_{23}^d \epsilon_{ij} F_2^i H^j F_3^d + y_{23}^u \delta_{ij} F_2^i (H^\dagger)^j F_3^u - M_{12} \epsilon_{ij} F_1^i F_2^j + M_{34}^u F_3^u F_4^d - M_{34}^d F_3^d F_4^u + \text{h.c.}$$

Vector-like